Real-Time Scheduling Theory
and Ada

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Abstract: The Ada tasking model was intended to support the management of concurrency in a priority-driven scheduling environment. In this paper, we review some important results of a priority-based scheduling theory, illustrate its applications with examples, discuss its implications for the Ada tasking model, and suggest workarounds that permit us to implement analytical scheduling algorithms within the existing framework of Ada. This paper is a revision of CMU/SEI-88-TR-33.1 A shortened version is also being presented at the 1989 Ada-Europe Conference.

1. Introduction

1.1. Background

The Real-Time Scheduling in Ada Project at the Software Engineering Institute is a cooperative effort between the SEI, Carnegie Mellon University, system developers in industry, Ada vendors, and DoD agencies. It aims at applying the scheduling theory reviewed in this paper to the design and implementation of hard real-time systems in Ada. The scheduling algorithms and theories developed under this project and at Carnegie Mellon provide an analytical basis for understanding the timing behavior of real-time systems. The project is implementing these scheduling algorithms in an Ada runtime system, and is coding examples of real-time systems to evaluate the suitability of the whole approach using a generic avionics application, a generic missile application, and a generic inertial navigation system. This paper summarizes some of the scheduling approaches being studied and shows how they can be applied in an Ada context.

Traditionally, many real-time systems use cyclical executives to schedule concurrent threads of execution. Under this approach, a programmer lays out an execution timeline by hand to serialize the execution of critical sections and to meet task deadlines. While such an approach is adequate for simple systems, it quickly becomes unmanageable for large systems. It is a painful process to develop application code so that the compiled segments fit into the time slots of a cyclical executive and to ensure that the critical sections of different tasks do not interleave. Forcing programmers to schedule tasks by fitting code segments on a timeline is no better than the outdated approach of managing memory by manual memory overlay. Such an approach often destroys program structure and results in real-time programs that are difficult to understand and maintain.

The Ada tasking model represents a fundamental departure from the cyclical executive model. Indeed, the dynamic preemption of tasks at runtime generates nondeterministic

1The most important revisions affect our discussion of aperiodic tasks and our analysis of how to support the priority ceiling protocol.
timelines that are at odds with the very idea of a fixed execution timeline. This nondeterminism seems to make it impossible to decide whether real-time deadlines will be met. However, Ada’s tasking concepts are well-suited to the analytic scheduling theories being considered in our project. In essence, these theories ensure that as long as the CPU utilization of all tasks lies below a certain bound and appropriate scheduling algorithms are used for the CPU and I/O processing, all tasks will meet their deadlines without knowing exactly when any given task will be running. Even if there is a transient overload, a fixed subset of critical tasks will still meet their deadlines as long as their CPU utilizations lie within the appropriate bound. The theories also deal with aperiodic processing requirements, mode changes, and jitter requirements. Applying these theories to Ada makes Ada tasking truly useful for real-time applications while also putting the development and maintenance of real-time systems on an analytic, engineering basis, making these systems easier to develop and maintain.

1.2. Controlling Priority Inversion

To put real-time scheduling on an analytical basis, systems must be built in a way that ensures that high-priority tasks are minimally delayed by lower priority tasks when both are contending for the same resources. Priority inversion occurs when the use of a resource by a low priority task delays the execution of a high priority task. Priority inversion occurs either when task priorities are incorrectly assigned or when they are not used correctly when allocating resources. One common mistake in priority scheduling is assigning priorities solely according to task importance.

Example 1: Suppose that \( \tau_1 \) and \( \tau_2 \) are periodic tasks with periods 100 and 10, respectively. Both of them are initiated at \( t = 0 \), and task \( \tau_1 \) is more important than task \( \tau_2 \). Assume that task \( \tau_1 \) requires 10 units of execution time and its first deadline is at \( t = 100 \), while task \( \tau_2 \) needs 1 unit of execution time with its first deadline at \( t = 10 \). If task \( \tau_1 \) is assigned higher scheduling priority because of its importance, task \( \tau_2 \) will miss its deadline unnecessarily even though the total processor utilization is only 0.2. Both tasks can meet their deadlines using the rate monotonic algorithm [Liu & Layland 73], which assigns higher priorities to tasks with shorter periods. In fact, many more new tasks can be added into the system by using the simple rate monotonic scheduling algorithm.

Although priority inversion is undesirable, it cannot be completely eliminated. For example, when a low priority task is in a critical region, the higher priority task that needs the shared data must wait. Nonetheless, the duration of priority inversion must be tightly bounded in order to ensure a high degree of responsiveness and schedulability. Controlling priority inversion is a system level problem. The tasking model, runtime support, program design, and hardware architecture should all be part of the solution, not part of the problem. For example, there is a serious priority inversion problem in some existing IEEE 802.5 token ring implementations. While there are 8 priority levels in the token arbitration, the queuing of message packets is FIFO, i.e., message priorities are ignored. As a result, when a high priority packet is behind a low priority packet, the high priority packet has to wait not only for
the lower priority packet to be transmitted, but also for the transmission of all medium priority packets in the network. The result is "hurry-up at the processor but miss the deadline at the communication network." Using FIFO queueing in a real-time system is a classical case of priority inversion and can lead to extremely poor schedulability. Priority assignments must be observed at every level of a system for all forms of resource allocation. Minimizing the duration of priority inversion is the key to meeting deadlines and keeping systems responsive to aperiodic events.

Chapter 2 reviews some of the important results in real-time scheduling theory. We begin with the problem of scheduling independent periodic tasks. Next, we address the issues of maintaining stability under transient overload and the problem of scheduling both periodic and aperiodic tasks. We conclude Chapter 2 by reviewing the problems of real-time synchronization. In Chapter 3, we review the Ada tasking scheduling model and suggest some workarounds that permit us to implement many of the scheduling algorithms within the framework of existing Ada rules. Finally, we conclude this paper in Chapter 4.
2. Scheduling Real-Time Tasks

In this chapter, we provide an overview of some of the important issues of a real-time scheduling theory. We will begin with the problem of ensuring that independent periodic tasks meet their deadlines. Next, we show how to ensure that critical tasks meet their deadlines even when a system is temporarily overloaded. We then address the problem of scheduling both periodic and aperiodic tasks. Finally, we conclude this chapter by reviewing the problems of real-time synchronization and communication.

2.1. Periodic Tasks

Tasks are independent if their executions need not be synchronized. Given a set of independent periodic tasks, the rate monotonic scheduling algorithm gives each task a fixed priority and assigns higher priorities to tasks with shorter periods. A task set is said to be schedulable if all its deadlines are met, i.e., if every periodic task finishes its execution before the end of its period. Any set of independent periodic tasks is schedulable by the rate monotonic algorithm if the condition of Theorem 1 is met [Liu & Layland 73].

\[ \sum \frac{C_i}{T_i} \leq n \left( \frac{1}{n} \right) = U(n) \]

where \( C_i \) and \( T_i \) are the execution time and period of task \( \tau_i \) respectively.

Theorem 1 offers a sufficient (worst-case) condition that characterizes the schedulability of the rate monotonic algorithm. This bound converges to 69\% \( (\ln 2) \) as the number of tasks approaches infinity. Table 2-1 shows values of the bound for one to nine tasks.

<table>
<thead>
<tr>
<th>( U(n) )</th>
<th>1.0</th>
<th>0.756</th>
<th>0.728</th>
</tr>
</thead>
<tbody>
<tr>
<td>( n )</td>
<td>1</td>
<td>4</td>
<td>7</td>
</tr>
<tr>
<td>( U(n) )</td>
<td>0.828</td>
<td>0.743</td>
<td>0.724</td>
</tr>
<tr>
<td>( n )</td>
<td>2</td>
<td>5</td>
<td>8</td>
</tr>
<tr>
<td>( U(n) )</td>
<td>0.779</td>
<td>0.734</td>
<td>0.720</td>
</tr>
<tr>
<td>( n )</td>
<td>3</td>
<td>6</td>
<td>9</td>
</tr>
</tbody>
</table>

Table 2-1: Scheduling Bounds for One to Nine Independent Tasks

The bound of Theorem 1 is very pessimistic because the worst-case task set is contrived and unlikely to be encountered in practice. For a randomly chosen task set, the likely bound is 88\% [Lehoczky et al. 87]. To know if a set of given tasks with utilization greater than the bound of Theorem 1 can meet its deadlines, the conditions of Theorem 2 must be checked [Lehoczky et al. 87].

\[ \sum \frac{C_i}{T_i} \leq n \left( \frac{1}{n} \right) = U(n) \]

Theorem 2: A set of \( n \) independent periodic tasks scheduled by the rate monotonic algorithm will always meet its deadlines, for all task phasings, if and only if
∀ i, 1 ≤ i ≤ n, \( \min_{(k, l) \in R_i} \sum_{j=1}^{i} C_j \frac{l T_k}{T_j} \leq 1 \)

where \( C_j \) and \( T_j \) are the execution time and period of task \( \tau_j \) respectively and \( R_i = \{(k, l) \mid 1 \leq k \leq i, l = 1, \ldots, \lceil T_i/T_k \rceil \} \).

This theorem provides the exact schedulability criterion for independent periodic task sets under the rate monotonic algorithm. In effect, the theorem checks if each task can complete its execution before its first deadline by checking all the scheduling points. The scheduling points for task \( \tau \) are \( \tau \)'s first deadline and the ends of periods of higher priority tasks within \( \tau \)'s first deadline. In the formula, \( i \) denotes the task to be checked and \( k \) denotes each of the tasks that affects the completion time of task \( i \), i.e., task \( i \) and the higher priority tasks. For a given \( i \) and \( k \), each value of \( l \) represents the scheduling points of task \( k \). For example, suppose that we have tasks \( \tau_1 \) and \( \tau_2 \) with periods \( T_1 = 5 \) and \( T_2 = 14 \). For task \( \tau_1 \) (\( i = 1 \)) we have only one scheduling point, the end of task \( \tau_1 \)'s first period, i.e., \( i = k = 1 \) and \( (l = 1, \ldots, \lceil T_i/T_k \rceil = \lceil T_1/T_1 \rceil = 1) \). The scheduling point is, of course, \( \tau_1 \)'s first deadline \( (l T_k = 5, l = 1, k = 1) \). For task \( \tau_2 \) (\( i = 2 \)), there are two scheduling points from all higher priority tasks, \( \tau_k \) (\( k = 1 \)), i.e., \( (l = 1, \ldots, \lceil T_i/T_k \rceil = \lceil T_2/T_1 \rceil = 2) \). The two scheduling points are, of course, the two end points of task \( \tau_1 \)'s period within the first deadline of task \( \tau_2 \) at 14, i.e., \( (l T_k = 5, l = 1, k = 1) \) and \( (l T_k = 10, l = 2, k = 1) \). Finally, there is the scheduling point from \( \tau_2 \)'s own first deadline, i.e., \( (l T_k = 14, l = 1, k = 2) \). At each scheduling point, we check if the task in question can complete its execution at or before the scheduling point. This is illustrated in detail by Examples 3 and 8 below.

**Example 2:** Consider the case of three periodic tasks, where \( U_i = C_i/T_i \):

- Task \( \tau_1 \): \( C_1 = 20 \); \( T_1 = 100 \); \( U_1 = 0.2 \)
- Task \( \tau_2 \): \( C_2 = 40 \); \( T_2 = 150 \); \( U_2 = 0.267 \)
- Task \( \tau_3 \): \( C_3 = 100 \); \( T_3 = 350 \); \( U_3 = 0.286 \)

The total utilization of these three tasks is 0.753, which is below Theorem 1’s bound for three tasks: \( 3(2^{1/3} - 1) = 0.779 \). Hence, we know these three tasks are schedulable, i.e., they will meet their deadlines if \( \tau_1 \) is given the highest priority, \( \tau_2 \) the next highest, and \( \tau_3 \) the lowest.

The remaining 24.7% processor capacity can be used for low priority background processing. However, we can also use it for additional hard real-time computation.

**Example 3:** Suppose we replace \( \tau_1 \)'s algorithm with one that is more accurate and computationally intensive. Suppose the new algorithm doubles \( \tau_1 \)'s computation

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\(^2\)It was shown in [Liu & Layland 73] that when all the tasks are initiated at the same time (the worst-case phasing), if a task completes its execution before the end of its first period, it will never miss a deadline.
time from 20 to 40, so the total processor utilization increases from 0.753 to 0.953. Since the utilization of the first two tasks is 0.667, which is below Theorem 1’s bound for two tasks, $2(2^{1/2} - 1) = 0.828$, the first two tasks cannot miss their deadlines. For task $\tau_3$, we use Theorem 2 to check whether the task set is schedulable, i.e., we set $i = n = 3$, and check whether one of the following equations holds:

$$\forall k, l, 1 \leq k, l \leq 3, \quad \sum_{j=1}^{3} \left\lfloor \frac{LT_k}{T_j} \right\rfloor C_j \leq lT_k$$

To check if task $\tau_3$ can meet its deadline, it is only necessary to check the equation for values of $l$ and $k$ such that $lT_k \leq T_3 = 350$. If one of the equations is satisfied, the task set is schedulable.

$$C_1 + C_2 + C_3 \leq T_1 \quad 40 + 40 + 100 > 100 \quad l = 1, k = 1$$

or

$$2C_1 + C_2 + C_3 \leq T_2 \quad 80 + 40 + 100 > 150 \quad l = 1, k = 2$$

or

$$2C_1 + 2C_2 + C_3 \leq 2T_1 \quad 80 + 80 + 100 > 200 \quad l = 2, k = 1$$

or

$$3C_1 + 2C_2 + C_3 \leq 2T_2 \quad 120 + 80 + 100 = 300 \quad l = 2, k = 2, \text{ or } l = 3, k = 1$$

or

$$4C_1 + 3C_2 + C_3 \leq T_3 \quad 160 + 120 + 100 > 350 \quad l = 1, k = 3$$

The analysis shows that task $\tau_3$ is also schedulable and in the worst-case phasing will meet its deadline exactly at time 300. Hence, we can double the utilization of the first task from 20% to 40% and still meet all the deadlines. The remaining 4.7% processor capacity can be used for either background processing or a fourth hard deadline task, which has a period longer than that of $\tau_3$ and which satisfies the condition of Theorem 2.

A major advantage of using the rate monotonic algorithm is that it allows us to separate logical correctness concerns from timing correctness concerns. Suppose that a cyclical executive is used for this example. The major cycle must be the least common multiple of the task periods. In this example, the task periods are in the ratio 100:150:350 = 2:3:7. A minor cycle of 50 units would induce a major cycle of 42 minor cycles, which is an overly complex design. To reduce the number of minor cycles, we can try to modify the periods. For example, it might be possible to reduce the period of the longest task, from 350 to 300. The total utilization is then exactly 100%, and the period ratios are 2:3:6; the major cycle can then be 6 minor cycles of 50 units. To implement this approach and minimize the splitting of computations belonging to a single task, we could split task $\tau_1$ into two parts of 20 units

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3That is, after 300 units of time, $\tau_1$ will have run three times, $\tau_2$ will have run twice, and $\tau_3$ will have run once. The required amount of computation just fits within the allowed time, so each task meets its deadline. [Liu & Layland 73] showed that since the tasks meet their deadlines at least once within the period $T_3$, they will always meet their deadlines.

4Task $\tau_3$ just meets its deadline at 300 and hence we cannot add a task with a priority higher than that of task $\tau_3$. 

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computation each. The computation of task \( \tau_2 \) similarly could be split into at least two parts such that task \( \tau_3 \) need only be split into four parts. A possible timeline indicating the amount of computation for each task in each minor cycle is shown in the following table, where \( 20_1 \) on the first line indicates the first part of task \( \tau_1 \)'s computation, which takes 20 units of time.

| Cyclic Timeline for Example 3 |
|---|---|---|---|---|---|
|   | 1 | 2 | 3 | 4 | 5 | 6 |
| \( \tau_1 \) | 20_1 | 20_2 | 20_1 | 20_2 | 20_1 | 20_2 |
| \( \tau_2 \) | 30_1 | 10_2 | 30_1 | 10_2 |
| \( \tau_3 \) | 20_1 | 30_2 | 20_3 | 30_4 |

**Table 2-2:** Minor Cycle Timeline: Each Minor Cycle Is 50.

When processor utilization level is high and there are many tasks, fitting code segments into time slots can be a time-consuming iterative process. In addition, a later modification of any task may overflow a particular minor cycle and require the entire timeline to be redone. But more important, the cyclic executive approach has required us to modify the period of one of the tasks, increasing the utilization to 100% *without in fact doing more useful work*. Under the rate monotonic approach for this example, all deadlines are met, but total machine utilization must be 95.3% or less instead of 100% or less. This doesn’t mean the rate monotonic approach is less efficient. The capacity that isn’t needed to service real-time tasks in the rate monotonic approach can be used by background tasks, e.g., for built-in-test purposes. With the cyclic executive approach, no such additional work can be done in this example. Of course, the scheduling overhead for task preemption needs to be taken into account. If \( S \) is the amount of time needed for a single scheduling action, then the total utilization devoted to scheduling is \( 2S/T_1 + 2S/T_2 + 2S/T_3 \) since there are two scheduling actions per task. In the cyclic case, the scheduling overhead is partly in the (very small) time needed to dispatch each task’s code segment in each minor cycle and partly in the utilization wasted by decreasing the period for task 3. For this example, the scheduling overhead for the cyclic approach is at least 4.7%:

\[
\text{Actual Utilization} - \text{Required Utilization}, \text{ i.e.,}
100/300 - 100/350 = .333 - .286 = .047
\]

Thus, although the rate monotonic approach may seem to yield a lower maximum utilization than the cyclic approach, in practice, the cyclic approach may simply be consuming more machine time because the periods have been artificially shortened. In addition, cyclic executives get complicated when they have to deal with aperiodic events. The rate monotonic approach, as will be discussed later, readily accommodates aperiodic processing. Finally, the rate monotonic utilization bound as computed by Theorem 2 is a function of the periods and computation times of the task set. The utilization bound can always be increased by transforming task periods, as described in the next section.
2.2. Stability Under Transient Overload

In the previous section, the computation time of a task is assumed to be constant. However, in many applications, task execution times are stochastic, and the worst-case execution time can be significantly larger than the average execution time. To have a reasonably high average processor utilization, we must deal with the problem of transient overload. During transient overload, we want to guarantee the deadlines of the tasks that are critical to the mission, even at the cost of missing the deadlines of non-critical tasks.

We consider a scheduling algorithm to be stable if there is a fixed set of tasks (the stable set) that always meet their deadlines even if the processor is overloaded. Tasks outside the stable set may miss their deadlines as the processor load increases or as task phasings change. To ensure that the critical tasks always meet their deadlines, it is sufficient for them to belong to the stable set of tasks.

The rate monotonic algorithm is a stable scheduling algorithm. Suppose there are \( n \) tasks. The stable set consists of the first \( k \) highest priority tasks that satisfy Theorem 1 or 2. For example, if the utilization of the first \( k \) tasks is less than 69.3%, the tasks will always meet their deadlines no matter what the total processor load is. Of course, which tasks are in the stable task set depends on the worst-case utilizations of the particular tasks considered. The important point is that the rate monotonic algorithm guarantees that if such a set exists, it always consists of tasks with the highest priorities. This means that if a transient overload should develop, tasks with longer periods will miss their deadlines.

Of course, a task with a longer period could be more critical to an application than a task with a shorter period. One might attempt to ensure that the critical task always meets its deadline by assigning priorities according to a task’s importance. However, this approach can lead to poor schedulability. That is, with this approach, deadlines of critical tasks might be met only when the total utilization is low.

The period transformation technique can be used to ensure high utilization while meeting the deadline of an important, long-period task. Period transformation means turning a long-period important task into a high priority task by splitting its work over several short periods. For example, suppose task \( \tau \) with a long period \( T \) is not in the critical task set and must never miss its deadline. We can make \( \tau \) simulate a short period task by giving it a period of \( T/2 \) and suspending it after it executes half its worst-case execution time, \( C/2 \). The task is then resumed and finishes its work in the next execution period. It still completes its total computation before the end of period \( T \). From the viewpoint of the rate monotonic theory, the transformed task has the same utilization but a shorter period, \( T/2 \), and its priority is raised accordingly. It is important to note that the most important task need not have the shortest period. We only need to make sure that it is among the first \( n \) high priority tasks whose worst-case utilization is within the scheduling bound. A systematic procedure for period transformation with minimal task partitioning can be found in [Sha et al. 86].

Period transformation allows important tasks to have higher priority while keeping priority
assignments consistent with rate monotonic rules. This kind of transformation should be familiar to users of cyclic executives. The difference here is that we don’t need to adjust the code segment sizes so different code segments fit into shared time slots. Instead, τ simply requests suspension after performing $C/2$ amount of work. Alternatively, the runtime scheduler can be instructed to suspend the task after a certain amount of computation has been done, without affecting the application code.\(^5\)

The period transformation approach has another benefit — it can raise the rate monotonic utilization bound. Suppose the rate monotonic utilization bound is $U_{\text{max}} \leq 100\%$, i.e., total task utilization cannot be increased above $U_{\text{max}}$ without missing a deadline. When a period transformation is applied to the task set, $U_{\text{max}}$ will rise. For example:

**Example 4:** Let

- Task $\tau_1$: $C_1 = 4$; $T_1 = 10$; $U_1 = .400$
- Task $\tau_2$: $C_2 = 6$; $T_2 = 14$; $U_2 = .428$

The total utilization is .828, which just equals the bound of Theorem 2, so this set of two tasks is schedulable. If we apply Theorem 2, we find:

$$C_1 + C_2 \leq T_1 \quad 4 + 6 = 10 \quad l = 1, k = 1$$

or

$$2C_1 + C_2 \leq T_2 \quad 8 + 6 = 14 \quad l = 1, k = 2$$

So Theorem 2 says the task set is just schedulable. Now suppose we perform a period transformation on task $\tau_1$, so $C_1' = 2$ and $T_1' = 5$. The total utilization is the same and the set is still schedulable, but when we apply Theorem 2 we find:

$$C_1 + C_2 \leq T_1 \quad 2 + 6 \leq 5 \quad l = 1, k = 1$$

or

$$2C_1 + C_2 \leq 2T_1 \quad 4 + 6 \leq 10 \quad l = 2, k = 1$$

or

$$3C_1 + C_2 < T_2 \quad 6 + 6 < 14 \quad l = 1, k = 2$$

The third equation shows that the compute times for tasks $\tau_1$ and/or $\tau_2$ can be increased without violating the constraint. For example, the compute time of task $\tau_1$ can be increased by 2/3 units to 2.667, giving an overall schedulable utilization of $2.667/5 + 6/14 = .961$, or the compute time of Task $\tau_2$ can be increased to 8, giving an overall schedulable utilization of $2/5 + 8/14 = .971$. So the effect of the period transformation has been to raise the utilization bound from .828 to at least .961 and at most .971.

If periods are uniformly harmonic, i.e., if each period is an integral multiple of each shorter

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\(^5\)The scheduler must ensure that τ is not suspended while in a critical region since such a suspension can cause other tasks to miss their deadlines. If the suspension time arrives but the task is in a critical region, then the suspension should be delayed until the task exits the critical region. To account for this effect on the schedulability of the task set, the worst-case execution time must be increased by $\varepsilon$, the extra time spent in the critical region, i.e., τ’s utilization becomes $(0.5C+\varepsilon)/0.5T$.\(^5\)
period, the utilization bound of the rate monotonic algorithm is 100%\(^6\). So the utilization bound produced by the rate monotonic approach is only an upper bound on what can be achieved if the periods are not transformed. Of course, as the periods get shorter, the scheduling overhead utilization increases, so the amount of useful work that can be done decreases. For example, before a period transformation, the utilization for a task, including scheduling overhead, is \((C + 2S)/T\). After splitting the period into two parts, the utilization is \((.5C + 2S)/.5T\), so scheduling overhead is a larger part of the total utilization. However, the utilization bound is also increased in general. If the increase in utilization caused by the scheduling overhead is less than the increase in the utilization bound, then the period transformation is a win — more useful work can be done while meeting all deadlines.

2.3. Scheduling Both Aperiodic and Periodic Tasks

It is important to meet the regular deadlines of periodic tasks and the response time requirements of aperiodic events. ("Aperiodic tasks" are used to service such events.) Let us begin with a simple example.

**Example 5:** Suppose that we have two tasks. Let \(\tau_1\) be a periodic task with period 100 and execution time 99. Let \(\tau_2\) be an aperiodic task that appears once within a period of 100 but the arrival time is random. The execution time of task \(\tau_2\) is one unit. If we let the aperiodic task wait for the periodic task, then the average response time is about 50 units. The same can be said for a polling server, which provides one unit of service time in a period of 100. On the other hand, we can deposit one unit of service time in a "ticket box" every 100 units of time; when a new "ticket" is deposited, the unused old tickets, if any, are discarded. With this approach, no matter when the aperiodic event arrives during a period of 100, it will find there is a ticket for one unit of execution time at the ticket-box. That is, \(\tau_2\) can use the ticket to preempt \(\tau_1\) and execute immediately when the event occurs. In this case, \(\tau_2\)'s response time is precisely one unit and the deadlines of \(\tau_1\) are still guaranteed. This is the idea behind the deferrable server algorithm [Lehoczky 87], which reduces aperiodic response time by a factor of about 50 in this example.

In reality, there can be many periodic tasks whose periods can be arbitrary. Furthermore, aperiodic arrivals can be very bursty, as for a Poisson process. However, the idea remains unchanged. We should allow the aperiodic tasks to preempt the periodic tasks subject to not causing their deadlines to be missed. It was shown in [Lehoczky 87] that the deadlines of periodic tasks can be guaranteed provided that during a period of \(T_a\) units of time, there are no more than \(C_a\) units of time in which aperiodic tasks preempt periodic tasks. In addition, the total periodic and aperiodic utilization must be kept below \((U_a + \ln((2 + U_a)/(2U_a + 1)))\), where \(U_a = C_a/T_a\). And the server's period must observe the inequality \("T_a \leq (T - C_a)"\), where \(T\) is the period of a periodic task whose priority is next to the server.

\(^6\)For example, by transforming the periods in Example 3 so \(\tau_1'\) and \(\tau_2'\) both have periods of 50, the utilization bound is 100%, i.e., 4.7% more work can be done without missing a deadline.
Compared with background service, the deferrable server algorithm typically improves aperiodic response time by a factor between 2 and 10 [Lehoczky 87]. Under the deferrable server algorithm, both periodic and aperiodic task modules can be modified at will as long as the utilization bound is observed.

A variation of the deferrable server algorithm is known as the *sporadic* server algorithm [Sprunt et al. 89]. As for the deferrable server algorithm, we allocate $C_a$ units of computation time within a period of $T_a$ units of time. However, the $C_a$ of the server's budget is not refreshed until the budget is consumed. From a capacity planning point of view, a sporadic server is equivalent to a periodic task that performs polling. That is, we can place sporadic servers at various priority levels and use only Theorems 1 and 2 to perform a schedulability analysis. Sporadic and deferrable servers have similar performance gains over polling because any time an aperiodic task arrives, it can use the allocated budget immediately. When polling is used, however, an aperiodic arrival generally needs to wait for the next instant of polling. The sporadic server has the least runtime overhead. Both the polling and the deferrable servers have to be serviced periodically, even if there are no aperiodic arrivals. There is no overhead for the sporadic server until its execution budget has been consumed. In particular, there is no overhead if there are no aperiodic arrivals. Therefore, the sporadic server is especially suitable for handling emergency aperiodic events that occur rarely but must be responded to quickly.

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**Figure 2-1**: Scheduling Both Aperiodic and Periodic Tasks

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7 Early refreshing is also possible under certain conditions. See [Sprunt et al. 89].

8 The ticket box must be refreshed at the end of each deferrable server's period.
Simulation studies of the sporadic server algorithm [Sprunt et al. 89] show that in a lightly loaded system, aperiodic events are served 5-10 times faster than with background service, and 3-6 times faster than with polling. Figure 2-1, from [Sprunt et al. 89], shows one example of the relative performance between background execution, the deferrable server algorithm (DS), the sporadic server algorithm (SS), polling, and another algorithm, not explained here, called the priority exchange algorithm (PE). The analysis underlying these results assumes a Poisson arrival process with exponentially distributed service time. In addition, each server (other than the background server) is given a period that allows it to execute as the highest priority task. Aperiodic requests can therefore preempt the execution of periodic tasks as long as server execution time is available.

The maximum amount of aperiodic service time allowed before periodic tasks will miss their deadline is called the maximum server size. In this example, aperiodic tasks can preempt periodic tasks for at most 56.3% of the sporadic or polling server’s period without causing the deadlines of periodic tasks to be missed. For the deferrable server, only a smaller amount of service time is possible: 43.6%. In either case, the server is not allowed to execute at its assigned priority once its computation budget is exhausted, although it can continue to execute at background priority if time is available. A server’s budget is refreshed at the end of its period, at which time execution can resume at the server’s assigned priority. A server can resume its execution at its assigned priority, only when its budget is refreshed.

Figure 2-1 shows the average response times of the different scheduling algorithms as a function of average aperiodic workload. When the average aperiodic workload is small compared with the sporadic server size, randomly arriving requests are likely to find the server available and can successfully preempt the periodic tasks. This results in good performance. For example, when the average aperiodic workload is 5%, the deferrable and sporadic server response time is about 10% of the average background response time, while the average polling response time is about 65% of background response time. (This means the sporadic server gives about 6 times faster response than polling and 10 times faster than background service.) When the aperiodic workload increases, the likelihood of server availability decreases and the resulting performance advantage also decreases. For example, when the aperiodic load is 55%, the different server algorithms do not give significant performance improvement over background service.

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9 This means each server’s period must not be greater than the shortest period of all the periodic tasks. The sporadic server and polling server can have a period equal to that of the shortest period task. As mentioned earlier in this section, however, the deferrable server must have an even shorter period.

10 A 5% average aperiodic workload means that in the long run, the aperiodic requests consume about 5% of the CPU cycles, although the number of requests and their execution time vary from period to period and from request to request.
2.4. Task Synchronization

In the previous sections we have discussed the scheduling of independent tasks. Tasks, however, do interact. In this section, we will discuss how the rate monotonic scheduling theory can be applied to real-time tasks that must interact. The discussion is limited in this paper to scheduling within a uniprocessor. Readers who are interested in the multiprocessor synchronization problem should see [Rajkumar et al. 88].

Common synchronization primitives include semaphores, locks, monitors, and Ada rendezvous. Although the use of these or equivalent methods is necessary to protect the consistency of shared data or to guarantee the proper use of nonpreemptable resources, their use may jeopardize the ability of the system to meet its timing requirements. In fact, a direct application of these synchronization mechanisms may lead to an indefinite period of priority inversion and low schedulability.

Example 6: Suppose $J_1$, $J_2$, and $J_3$ are three jobs arranged in descending order of priority with $J_1$ having the highest priority. We assume that jobs $J_1$ and $J_3$ share a data structure guarded by a binary semaphore $S$. Suppose that at time $t_1$, job $J_3$ locks the semaphore $S$ and executes its critical section. During the execution of the critical section of job $J_3$, the high priority job $J_1$ is initiated, preempts $J_3$ and later attempts to use the shared data. However, job $J_1$ will be blocked on the semaphore $S$. We would hope that $J_1$, being the highest priority job, is blocked no longer than the time for job $J_3$ to complete its critical section. However, the duration of blocking is, in fact, unpredictable. This is because job $J_3$ can be preempted by the intermediate priority job $J_2$. The blocking of $J_3$, and hence that of $J_1$, will continue until $J_2$ and any other pending intermediate jobs are completed.

The blocking period in this example can be arbitrarily long. This situation can be partially remedied if a job in its critical section is not allowed to be preempted; however, this solution is only appropriate for very short critical sections because it creates unnecessary blocking. For instance, once a low priority job enters a long critical section, a high priority job that does not access the shared data structure may be needlessly blocked.

The **priority ceiling protocol** is a real-time synchronization protocol with two important properties: 1) freedom from mutual deadlock, and 2) bounded blocking, which means that at most one lower priority task can block a higher priority task [Goodenough & Sha 88, Sha et al. 87]. There are two ideas in the design of this protocol. First is the concept of priority inheritance: when a task $\tau$ blocks the execution of higher priority tasks, task $\tau$ executes at the highest priority level of all the tasks blocked by $\tau$. Secondly, we must guarantee that a critical section is allowed to start execution only if the section will always execute at a priority level that is higher than the (inherited) priority levels of any preempted critical sections. It was shown in [Sha et al. 87] that such a prioritized total ordering in the execution of critical sections leads to the two desired properties. To achieve such prioritized total ordering, we define the **priority ceiling** of a binary semaphore $S$ to be the highest priority of all tasks that may lock $S$. When a task $\tau$ attempts to execute one of its critical sections, it will be suspended unless its priority is higher than the priority ceilings of all semaphores currently
locked by tasks other than \( \tau \). If task \( \tau \) is unable to enter its critical section for this reason, the task that holds the lock on the semaphore with the highest priority ceiling is said to be blocking \( \tau \) and hence inherits the priority of \( \tau \). As long as a task \( \tau \) is not attempting to enter one of its critical sections, it will preempt every task that has a lower priority.

**Example 7:** Suppose that we have two jobs \( J_1 \) and \( J_2 \) in the system. In addition, there are two shared data structures protected by binary semaphores \( S_1 \) and \( S_2 \) respectively. Suppose the sequence of processing steps for each job is as follows.

\[
J_1 = \{ \cdots, P(S_1), \cdots, P(S_2), \cdots, V(S_2), \cdots \}
\]

\[
J_2 = \{ \cdots, P(S_2), \cdots, P(S_1), \cdots, V(S_1), \cdots \}
\]

Recall that the priority of job \( J_1 \) is assumed to be higher than that of job \( J_2 \). Thus, the priority ceilings of both semaphores \( S_1 \) and \( S_2 \) are equal to the priority of job \( J_1 \). Suppose that at time \( t_0 \), \( J_2 \) is initiated and it begins execution and then locks semaphore \( S_2 \). At time \( t_1 \), job \( J_1 \) is initiated and preempts job \( J_2 \) and at time \( t_2 \), job \( J_1 \) tries to enter its critical section by making an indivisible system call to execute \( P(S_1) \). However, the runtime system will find that the priority of \( J_1 \) is not higher than the priority ceiling of locked semaphore \( S_2 \). Hence, the runtime system suspends job \( J_1 \) without locking \( S_1 \). Job \( J_2 \) now inherits the priority of job \( J_1 \) and resumes execution. Note that \( J_1 \) is blocked outside its critical section. As \( J_1 \) is not given the lock on \( S_1 \) but suspended instead, the potential deadlock involving \( J_1 \) and \( J_2 \) is prevented. Once \( J_2 \) exits its critical section, it will return to its assigned priority and immediately be preempted by job \( J_1 \). From this point on, \( J_1 \) will execute to completion, and then \( J_2 \) will resume its execution until its completion.

Let \( B_i \) be the longest duration of blocking that can be experienced by task \( \tau_i \). The following two theorems indicate whether the deadlines of a set of periodic tasks can be met if the priority ceiling protocol is used.

**Theorem 3:** A set of \( n \) periodic tasks using the priority ceiling protocol can be scheduled by the rate monotonic algorithm, for all task phasings, if the following condition is satisfied [Sha et al. 87]:

\[
\frac{C_1}{T_1} + \cdots + \frac{C_n}{T_n} + \max \left( \frac{B_1}{T_1}, \cdots, \frac{B_{n-1}}{T_{n-1}} \right) \leq n(2^{1/n} - 1)
\]

**Theorem 4:** A set of \( n \) periodic tasks using the priority ceiling protocol can be scheduled by the rate monotonic algorithm for all task phasings if the following condition is satisfied [Sha et al. 87].

\[
\forall i, \ 1 \leq i \leq n, \quad \min_{(k, l) \in R_i} \left( \sum_{j=1}^{i-1} C_j \frac{1}{l T_k} \left[ \frac{l T_k}{T_j} \right] + C_i \frac{1}{l T_k} \left[ \frac{B_i}{l T_k} \right] \right) \leq 1
\]

where \( C_i, T_i, \) and \( R_i \) are defined in Theorem 2, and \( B_i \) is the worst-case blocking time for \( \tau_i \).
Remark: Theorems 3 and 4 generalize Theorems 1 and 2 by taking blocking into consideration. The $B_i$'s in Theorems 3 and 4 can be used to account for any delay caused by resource sharing. Note that the upper limit of the summation in the theorem is $(i - 1)$ instead of $i$, as in Theorem 2.

In the application of Theorems 3 and 4, it is important to realize that under the priority ceiling protocol, a task $\tau$ can be blocked by a lower priority task $\tau_L$ if $\tau_L$ may lock a semaphore $S$ whose priority ceiling is higher than or equal to the priority of task $\tau$, even if $\tau$ and $\tau_L$ do not share any semaphore. For example, suppose that $\tau_L$ locks $S$ first. Next, $\tau$ is initiated and preempts $\tau_L$. Later, a high priority task $\tau_H$ is initiated and attempts to lock $S$. Task $\tau_H$ will be blocked. Task $\tau_L$ now inherits the priority of $\tau_H$ and executes. Note that $\tau$ has to wait for the critical section of $\tau_L$ even though $\tau$ and $\tau_L$ do not share any semaphore. We call such blocking push-through blocking. Push-through blocking is the price for avoiding unbounded priority inversion. If task $\tau_L$ does not inherit the priority of $\tau_H$, task $\tau_H$ can be indirectly preempted by task $\tau$ and all the tasks that have priority higher than that of $\tau_L$. Finally, we want to point out that even if task $\tau_H$ does not attempt to lock $S$ but attempts to lock another unlocked semaphore, $\tau_H$ will still be blocked by the priority ceiling protocol because the priority of $\tau_H$ is not higher than the priority ceiling of $S$. We term this form of blocking as ceiling blocking. Ceiling block is the price for ensuring the freedom of deadlock and the property of a task being blocked at most once.

2.5. An Example Application of the Theory

In this section, we give a simple example to illustrate the application of the scheduling theory.

\textit{Example 8:} Consider the following task set.

1. Emergency handling task: execution time = 5 msec; worst case inter-arrival time = 50 msec; deadline is 6 msec after arrival.

2. Aperiodic event handling tasks: average execution time = 2 msec; average inter-arrival time = 40 msec; fast response time is desirable but there are no hard deadlines.

3. Periodic task $\tau_1$: execution time = 20 msec; period = 100 msec; deadline is at the end of each period.

    In addition, $\tau_3$ may block $\tau_1$ for 10 msec by using a shared communication server, and task $\tau_2$ may block $\tau_1$ for 20 msec by using a shared data object.

4. Periodic task $\tau_2$: execution time = 40 msec; period = 150 msec; deadline is 20 msec before the end of each period.
5. Periodic task $\tau_3$: execution time = 100 msec; period = 350 msec; deadline is at the end of each period.

Solution: First, we create a sporadic server for the emergency task, with a period of 50 msec and a service time 5 msec. Since the server has the shortest period, the rate monotonic algorithm will give this server the highest priority. It follows that the emergency task can meet its deadline.

Since the aperiodic tasks have no deadlines, they can be assigned a low background priority. However, since fast response time is desirable, we create a sporadic server executing at the second highest priority. The size of the server is a design issue. A larger server (i.e., a server with higher utilization) needs more processor cycles but will give better response time. In this example, we choose a large server with a period of 100 msec and a service time of 10 msec. We now have two tasks with a period of 100 msec, the aperiodic server and periodic task $\tau_1$. The rate monotonic algorithm allows us to break the tie arbitrarily, and we let the server have the higher priority.

We now have to check if the three periodic tasks can meet their deadlines. Since under the priority ceiling protocol a task can be blocked by lower priority tasks at most once, the maximal blocking time for task $\tau_1$ is $B_1 = \max(10, 20)$ msec = 20 msec. Since $\tau_3$ may lock the semaphore $S_c$ associated with the communication server and the priority ceiling of $S_c$ is higher than that of task $\tau_2$, task $\tau_2$ can be blocked by task $\tau_3$ for 10 msec.\(^{11}\) Finally, task $\tau_2$ has to finish 20 msec earlier than the nominal deadline for a periodic task. This is equivalent to saying that $\tau_2$ will always be blocked for an additional 20 msec but its deadline is at the end of the period. Hence, $B_2 = (10 + 20)$ msec = 30 msec.\(^{12}\) At this point, we can directly apply Theorem 4. However, we can also reduce the number of steps in the analysis by noting that period 50 and 100 are harmonics and we can treat the emergency server as if it has a period of 100 msec and a service time of 10 msec, instead of a period of 50 msec and a service time of 5 msec. We now have three tasks with a period of 100 msec and an execution time of 20 msec, 10 msec, and 10 msec respectively. For the purpose of analysis, these three tasks can be replaced by a single periodic task with a period of 100 msec and an execution time of 40 msec (20 + 10 + 10). We now have the following three equivalent periodic tasks for analysis:

- Task $\tau_1$: $C_1 = 40$; $T_1 = 100$; $B_1 = 20$; $U_1 = 0.4$
- Task $\tau_2$: $C_2 = 40$; $T_2 = 150$; $B_2 = 30$; $U_2 = 0.267$
- Task $\tau_3$: $C_3 = 100$; $T_3 = 350$; $B_3 = 0$; $U_3 = 0.286$

\(^{11}\)This may occur if $\tau_3$ blocks $\tau_1$ and inherits the priority of $\tau_1$.

\(^{12}\)Note that the blocked-at-most-once result does not apply here. It only applies to blocking caused by task synchronization using the priority ceiling protocol.
Using Theorem 4:

1. Task $\tau_1$: Check $C_1 + B_1 \leq T_1$. Since $40 + 20 \leq 100$, task $\tau_1$ is schedulable.

2. Task $\tau_2$: Check whether either

\begin{align*}
C_1 + C_2 + B_2 &\leq T_1 & \quad 40 + 40 + 30 &> 100 \\
2C_1 + C_2 + B_2 &\leq T_2 & \quad 80 + 40 + 30 &= 150
\end{align*}

Task $\tau_2$ is schedulable and in the worst-case phasing will meet its deadline exactly at time 150.

3. Task $\tau_3$: Check whether either

\begin{align*}
C_1 + C_2 + C_3 &\leq T_1 & \quad 40 + 40 + 100 &> 100 \\
2C_1 + C_2 + C_3 &\leq T_2 & \quad 80 + 40 + 100 &> 150 \\
2C_1 + 2C_2 + C_3 &\leq 2T_1 & \quad 80 + 80 + 100 &> 200 \\
3C_1 + 2C_2 + C_3 &\leq 2T_2 & \quad 120 + 80 + 100 &= 300 \\
4C_1 + 3C_2 + C_3 &\leq T_3 & \quad 160 + 120 + 100 &> 350
\end{align*}

Task $\tau_3$ is also schedulable and in the worst-case phasing will meet its deadline exactly at time 300. It follows that all three periodic tasks can meet their deadlines.

We now determine the response time of the aperiodics. The server capacity is 10% and the average aperiodic workload is 5% (2/40). Because most of the aperiodic arrivals can find "tickets," we would expect a good response time. Indeed, using a M/M/1 [Kleinrock 75] approximation for the lightly loaded server, the expected response time for the aperiodics is

\[ W = \frac{E[S]}{1 - \rho} = \frac{2}{1 - (0.05/0.10)} \approx 4 \text{ msec} \]

where $E[S]$ is the average execution time of aperiodic tasks and $\rho$ is the average server utilization. Finally, we want to point out that although the worst-case total periodic and server workload is 95%, we can still do quite a bit of background processing since the soft deadline aperiodics and the emergency task are unlikely to fully utilize the servers. The results derived for this example show how the scheduling theory puts real-time programming on an analytic engineering basis.
3. Real-Time Scheduling in Ada

Although Ada was intended for use in building real-time systems, its suitability for real-time programming has been widely questioned. Many of these questions concern practical issues, such as the cost of performing a rendezvous, minimizing the duration of interrupt masking in the runtime system, providing efficient support for interrupt handling, etc. These problems are being addressed by compiler vendors who are aiming at the real-time market. More important are concerns about the suitability of Ada’s conceptual model for dealing with real-time programming. For example, tasks in Ada run nondeterministically, making it hard for traditional real-time programmers to decide whether any tasks will meet their deadlines. In addition, the scheduling rules of Ada don’t seem to support prioritized scheduling well. Prioritized tasks are queued in FIFO order rather than by priority; high priority tasks can be delayed indefinitely when calling low priority tasks (due to priority inversion; see [Goodenough & Sha 88] for an example); and task priorities cannot be changed when application demands change at runtime. Fortunately, it appears that none of these problems presents insurmountable difficulties; solutions exist within the current language framework, although some language changes would be helpful to ensure uniform implementation support. The Real-Time Scheduling in Ada Project at the SEI is specifying coding guidelines and runtime system support needed to use analytic scheduling theory in Ada programs. The guidelines are still evolving and being evaluated; but so far, it seems likely they will meet the needs of a useful range of systems.

The rest of this section summarizes the approach being taken by the project, and then shows how Ada’s scheduling rules can be interpreted to support the requirements of rate monotonic scheduling algorithms.

3.1. Ada Real-Time Design Guidelines

The coding and design guidelines being developed by the SEI Real-Time Scheduling in Ada Project reflect a basic principle of real-time programming — write systems in a way that minimizes priority inversion; that is, minimize the time a high priority task has to wait for the execution of lower priority tasks.

For example, consider a set of periodic tasks, called clients, that must exchange data among themselves. They do not call each other to exchange data. Instead, whenever they must read or write shared data or send a message, they call a server task. Each server task has a simple structure — an endless loop with a single select statement with no guards.\(^{13}\) This structure models the notion of critical regions guarded by a semaphore; each entry is a critical region for the task calling the entry.

\(^{13}\)The prohibition against guards simplifies the schedulability analysis and the runtime system implementation, but otherwise is not essential.
Client tasks are assigned priorities according to rate monotonic principles; that is, tasks with the shortest periods are given the highest priorities. There are two options when assigning a priority to a server. If the Ada runtime system supports the priority ceiling protocol directly (the next section explains why the runtime system is allowed to support the protocol), then give the server a low or an undefined priority. In addition, tell the runtime system the priority ceiling of the server, i.e., the highest priority of all its clients. Then, when a server is executing on behalf of a client task, no other client task will be allowed to call any server unless the client’s priority is higher than the executing server’s priority ceiling. If the client does not have a sufficiently high priority, its call will be blocked (i.e., the caller will be preempted just before the call would be placed on an entry’s queue), and the server’s priority will be raised to the priority of the blocked caller. This treatment of competing calls will ensure that a high priority task is blocked at most once by a server [Goodenough & Sha 88]. Moreover, because calls are preempted before they are queued, when the executing server completes its rendezvous, the highest priority blocked task will be able to execute. In effect, calls will be processed in priority order, and mutual deadlock caused by nested server calls will be impossible.

If the priority ceiling protocol is not directly supported by an Ada runtime system, the effect of the protocol can often nonetheless be achieved. Suppose a server is used to synchronize access to data shared by several tasks. The ceiling protocol requires that while a server is in rendezvous with a client, no other server be called by any client unless the client has a priority higher than the server’s priority ceiling. This effect can be easily achieved using existing Ada runtime systems by assigning the server task a ceiling priority, i.e., a priority that is one greater than the priority of its highest priority caller. In this case, the server will be either waiting for a client or be serving a client at a priority that prevents other clients from executing and calling any server with an equal or lower priority ceiling. This approach avoids the prioritized queueing problem because there will never be more than one queued client task. It is also not hard to see why mutual deadlock will be impossible.

This simple approximation to the ceiling protocol will not work, however, if the server task suspends itself while in rendezvous. Such a suspension will allow client tasks with priorities lower than that of the suspended server to rendezvous with other servers; this violates the principle of the ceiling protocol. Such a suspension can be caused by either the server’s need to synchronize with external I/O events in a uniprocessor or the server’s need to rendezvous with another task in another processor in a multiprocessor. When a server task can suspend itself while in a rendezvous, care must be taken to ensure that no calls are accepted by other servers having the same or a lower ceiling priority. In addition, because queues can now develop, it is important that queued calls be serviced in priority order. Preventing inappropriate server calls and ensuring priority queueing can require quite complex code involving entry families, so this method of implementing the ceiling protocol is probably unsuitable when server tasks are allowed to suspend themselves.

From a schedulability viewpoint, the execution time spent in each rendezvous with a server that does not suspend itself is counted as part of the computing time, C_i, for the client task \( \tau_i \). Because the use of servers is a synchronization problem, Theorems 3 and 4 must be
used to account for blocking time. Under the priority ceiling protocol, the maximum blocking
time for a nonserver task at priority level $i$ is the longest entry call by a lower level client task
to a server whose priority ceiling is equal to or higher than the priority of $i$. (Note that this
definition of blocking time means that even if a task, $\tau$, makes no server calls, the
schedulability analysis for $\tau$ must include blocking time for lower priority client tasks; see
Example above.) If a server can suspend itself, it is sufficient to treat the suspension time
as server execution time in the schedulability analysis; work is underway to see when less
pessimistic treatment may be possible.

The worst-case blocking time for a client task is the same whether the ceiling protocol is
supported directly by the runtime system or is approximated by giving the server an appropria-
tely high priority. The essential difference between the direct implementation and the
approximation method is that the server priority will be raised only when necessary when the
direct implementation is used. This tends to generate less blocking on average and hence
better response time when aperiodic tasks have a priority below that of a server’s ceiling. In
addition, in transient overload situations, noncritical periodic tasks having a priority lower
than a server’s ceiling are less likely to miss their deadlines. In short, the direct implemen-
tation gives better average case behavior, especially when the entry calls are relatively long.
When the entry calls are short compared with client task execution times, the performance
difference is insignificant.

Despite the complications that arise when server tasks can suspend themselves, our point is
that the schedulability theory can be readily applied to Ada programs, ensuring that dead-
lines are met even when timelines are nondeterministic. In the next section we discuss how
to interpret Ada’s scheduling rules so that Ada runtime systems can support rate monotonic
scheduling algorithms directly.

### 3.2. On Ada Scheduling Rules

First of all, the Ada tasking model is well-suited, in principle, to the use of the analytic
scheduling theories presented in this paper. When using these theories, a programmer
doesn’t need to know when tasks are running to be sure that deadlines will be met. That is,
both Ada and the theory abstract away the details of an execution timeline. Although Ada
tasks fit well with the theory at the conceptual level, Ada and the theory differ on the rules for
determining when a task is eligible to run and its execution priority. For example, if a high
priority task calls a server task that is already in rendezvous with a low priority task, the
rendezvous can continue at the priority of the task being served instead of being increased
because a high priority task is waiting. Under these circumstances, the high priority task
can be blocked as long as there are medium priority jobs able to run. But there are a variety
of solutions to this problem. The most general solution within the constraints of the lan-
guage is simply to not use pragma PRIORITY at all. If all tasks in a program have no as-
signed priority, then the runtime system is free to use any convenient algorithm for deciding
which eligible task to run. An implementation-dependent pragma could be used to give
“scheduling priorities” to tasks, i.e., indications of scheduling importance that would be used
in accordance with rate monotonic scheduling algorithms. This approach would even allow "priorities" to be changed dynamically by the programmer because such changes only affect the scheduling of tasks that, in a legalistic sense, have no Ada priorities at all. The only problem with this approach is that entry calls are still queued in FIFO order rather than by priority. However, this problem can often be solved by using a coding style that prevents queues from having more than one task, making the FIFO issue irrelevant, or by suspending calling tasks just before they are queued. Of course, telling programmers to assign "scheduling priorities" to tasks but not to use pragma PRIORITY, surely says we are fighting the language rather than taking advantage of it. But the important point is that no official revisions to the language are needed to take advantage of the scheduling theories and algorithms described in this paper and being developed by our project. Here are the relevant Ada rules and appropriate ways of interpreting them:

- **CPU allocation:** priorities must be observed. Ada requires that the highest priority task eligible to run be given the CPU when this is "sensible." "Sensible" for a uniprocessor is usually interpreted to mean that if an entry call by an executing task can be accepted, the call must be accepted and no lower priority task can be allowed to execute. Although this interpretation may seem to be obviously the best, it is in fact not correct for the priority ceiling protocol, which gives better service to high priority tasks and avoids deadlock by blocking calls from medium priority tasks when certain low priority entry calls are already in progress. For example (see Figure 3-1), suppose a resource is guarded by a server task S, and suppose a low priority task is in rendezvous with S. Also suppose the priority ceiling of S is H (i.e., the highest priority task that can call S has priority H). Now suppose execution of the rendezvous is preempted by a medium priority task M, whose priority is less than H. Suppose M tries to call T, a server other than S. The priority ceiling protocol says that the call from M must be blocked, but the normal interpretation of Ada's scheduling rules would imply that the call of M must be accepted because M is the highest priority task that is eligible to run and T is able to accept the call.

Solution: There are several solutions to this problem. First of all, the priority ceiling protocol need not be applied to all called Ada tasks; it need only be applied to those tasks that are servers in the sense defined in the previous section. The simplest approach (in terms of living within the current interpretations of Ada rules) is to give each server task a unique priority that is higher than the priority of any task calling the server.

Another approach is to implement the priority ceiling protocol directly for server tasks. This approach is clearly allowed by Ada's scheduling rules as long as the server task is given no assigned priority. A server task with no assigned priority can preempt any other task at any time, because Ada's scheduling rules, in essence, do not specify how tasks with no priority are scheduled. For example, task S is allowed to preempt task M just at the point where M is about to call T. Moreover, when a server with no assigned priority is in a rendezvous with a client task, the Ada rules say the server is executed with "at least" the priority of the client task. Because the priority ceiling rules require that the server task be executed with the priority of the calling task or of any blocked task (whichever is higher), the Ada rule allows the server to be executed with the
(higher) priority of the blocked task. In the example, this means S, in effect, continues its rendezvous with M’s priority. When the rendezvous is complete, M’s call is allowed to succeed. Hence, the priority ceiling protocol can certainly be implemented directly when server tasks have no assigned priority.

Figure 3-1: Blocking Due to the Priority Ceiling Protocol

A more aggressive interpretation of Ada rules is to note that a high priority task must preempt a lower priority task only when it is "sensible" to do so. Surely it is not "sensible" for a medium priority task to start its rendezvous with T if the effect could be to delay the execution of a high priority task unnecessarily.14 In short, the priority ceiling protocol provides a set of rules saying when it is "sensible" to allow a higher priority task to run, and hence, these rules can be followed directly by the runtime system even when a server task has an assigned priority.

In short, there are several ways to argue that it is possible to support the priority ceiling protocol’s view of priorities within the current Ada rules.

• Hardware task priority: always higher than software task priorities. This Ada rule reflects current hardware designs, but hardware interrupts should not always have the highest priority from the viewpoint of the rate monotonic theory.

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14 Suppose S can call T (such a call models a nested critical region). If M’s rendezvous with T is allowed to start and H then calls S, H could be delayed until the completion of both S and T’s rendezvous; that is, H would be blocked by more than one lower priority task. This is one of the reasons for preventing M’s call.
Solution: When handling an interrupt that, in terms of the rate monotonic theory, should have a lower priority than the priority of some application task, keep the interrupt handling actions short (which is already a common practice) and include the interrupt handling duration as blocking time in the rate monotonic analysis. In other words, use the scheduling theory to take into account the effect of this source of priority inversion.

• Priority rules for task rendezvous:

  • **Selective wait:** priority can be ignored. That is, the scheduler is allowed, but not required, to take priorities into account when tasks of different priorities are waiting at open select alternatives.

    Solution: Because Ada allows, but does not require taking these priorities into account, ensure that the runtime system does use these priorities to decide which call to accept. Alternatively, if the priority ceiling protocol is used, there is never more than one waiting task to select.

  • **Called task priority:** only increased during rendezvous.

    Solution: Use the solutions discussed under “CPU allocation” above; that is, increase the priority of a server when the ceiling protocol says this is “sensible,” or give the called task no priority at all and use priority ceiling rules to say when the server is allowed to execute, or give servers a priority higher than that of their callers.

• **FIFO entry queues:** Ada requires that the priority of calling tasks be ignored; calls must be serviced in their order of arrival, not in order of priority. Using FIFO queues rather than prioritized queues usually has a serious negative effect on real-time schedulability. FIFO queues must be avoided.

    Solution: As noted earlier, often it is possible to avoid FIFO queueing by preventing queues from being formed at all. If the runtime system does not prevent queues from forming, then entry families can, of course, be used to get the effect of prioritized queueing.

• **Task priorities:** fixed. This rule is inappropriate when task priorities need to be changed at runtime. For example, when a new mode is initiated, the frequency of a task and/or its criticality may change, implying its priority must change. In addition, the scheduling rules for a certain class of aperiodic servers demand that the priority of such a server be lowered when it is about to exceed the maximum execution time allowed for a certain interval of time, and be raised when its service capacity has been restored [Sprunt et al. 89].

    Solution: When an application needs to adjust the priority of a task at runtime, this task should be declared as having no Ada priority. The runtime system can then be given a way of scheduling the task appropriately by, in effect, changing its priority.
From what our project has learned so far, it seems to be possible in practice to support analytic scheduling algorithms in Ada by using an enlightened interpretation of Ada’s scheduling rules together with a combination of runtime system modifications and appropriate coding guidelines. Of course, it would be better if the language did not get in the way of priority scheduling principles. The future revision of Ada should probably reword some of these rules so that priority-based scheduling is more clearly and uniformly supported.
4. Conclusion

Ada tasking was intended to be used for real-time programming. However, the Ada tasking model represents a fundamental departure from the traditional cyclical executive model. Indeed, the dynamic preemption of tasks at runtime generates nondeterministic timelines that are at odds with the very idea of a fixed execution timeline required by a cyclical executive.

In this paper, we have reviewed some important results of priority scheduling theory. Together with Ada tasking, they allow programmers to reason with confidence about timing correctness at the tasking level of abstraction. As long as analytic scheduling algorithms are supported by the runtime system and resource utilization bounds on CPU, I/O drivers, and communication media are observed, the timing constraints will be guaranteed. Even if there is a transient overload, the tasks missing deadlines will be in a predefined order.

Although the treatment of priorities by the current Ada tasking model can and should be improved, it seems that the scheduling algorithms can be used today within the existing Ada rules if an appropriate coding and design approach is taken, and if schedulers are written to take full advantage of certain coding styles and the existing flexibility in the scheduling rules. Additional reports on how this can be done are in preparation at the Software Engineering Institute.

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