

Design and Implementation of the GraphBLAS Template Library (GBTL)

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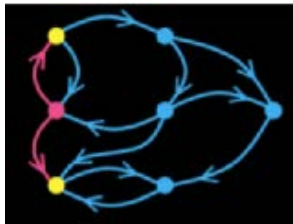
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What is this talk about?

- GraphBLAS
 - an emerging paradigm for graph computation
 - programs new graph algorithms in a highly abstract *language of linear algebra*.
 - executes in a wide variety of programming environments
- Our implementation of GraphBLAS
 - Graph BLAS Template Library (GBTL)
 - High-level C++ *frontend* (some features still in progress)
 - Algorithms written in terms of the API
 - Released at: <https://github.com/cmu-sei/gbtl>



Graph BLAS Forum
<http://www.graphblas.org>



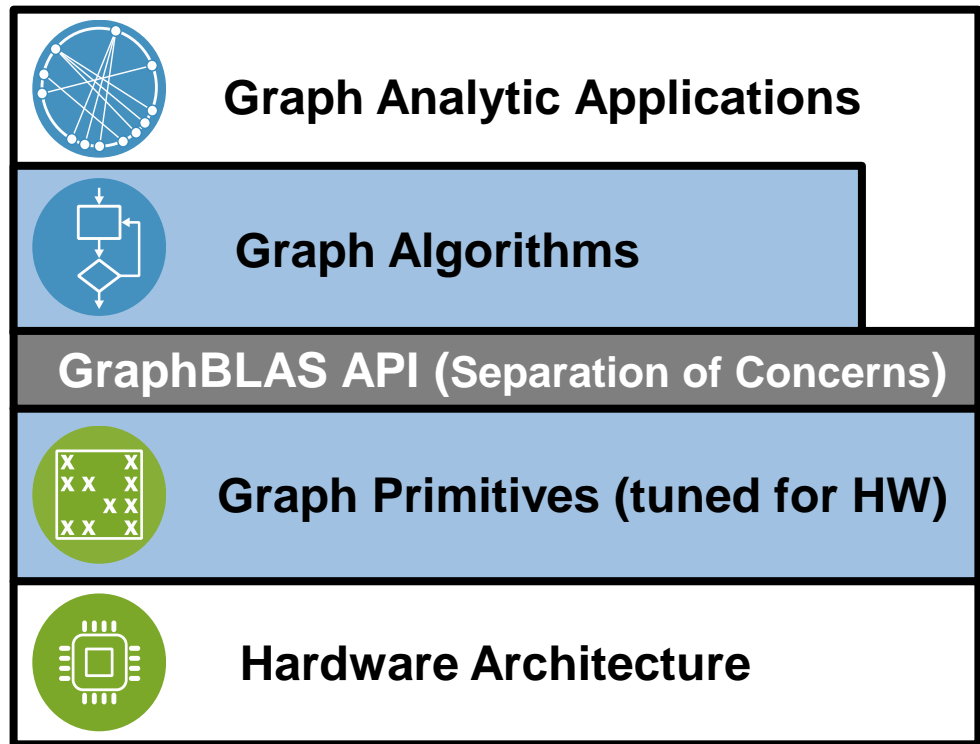
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Design Goals

- Separation of concerns:
 - Algorithm development
 - Hardware tuning
- Low overhead
- Support patterns for scalable, high-performance computing
- Templated C++ implementation
 - Allows for generic programming and template metaprogramming
 - Allows generic semantic checks (compile time)
- Similarity to the C API specification (under development)
- Easy to use



Contents

- Operations Overview
- Object Design
- Operations – Function Signatures
- Example Algorithm
- Summary and Future Work



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- **Operations Overview**
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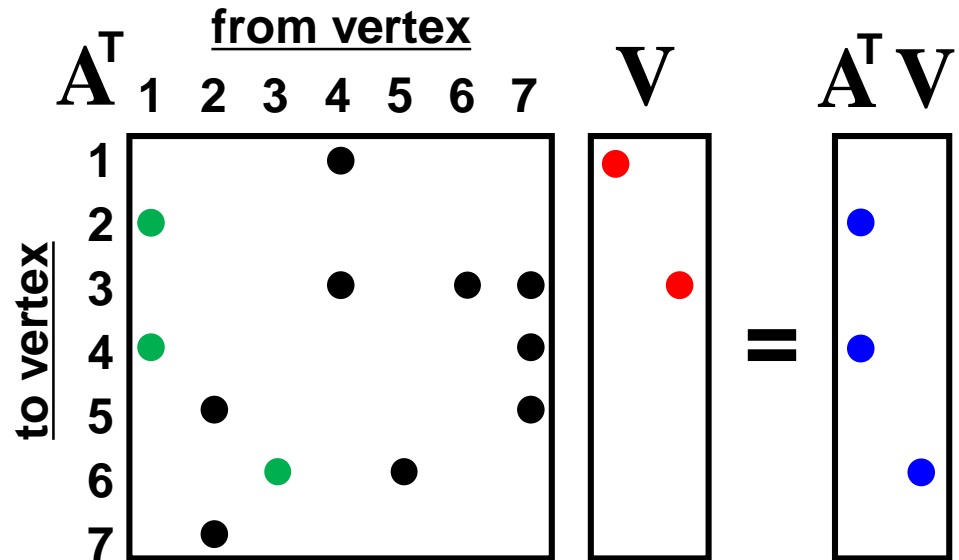
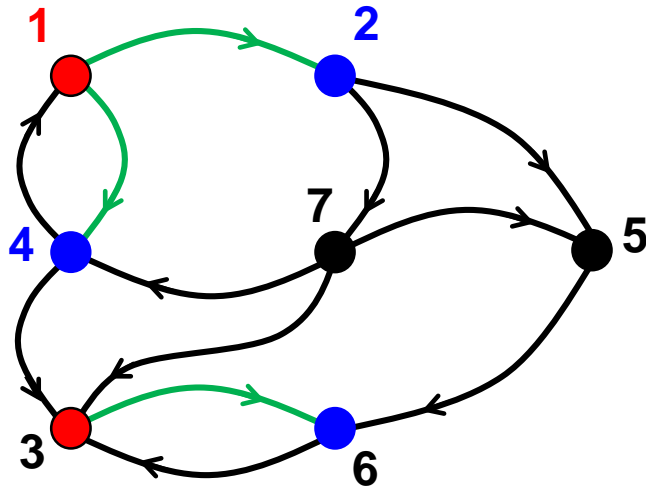
Background: GraphBLAS Operations

Operation	Description	Old Name
MxM, MxV, VxM	Perform matrix <i>multiplication</i> (e.g., BFS traversal)	SpGEMM
EwiseAdd, EwiseMult	Element-wise <i>addition</i> and <i>multiplication</i> of matrices (e.g., graph union, intersection)	SpEwiseX
Extract	Extract a sub-matrix from a larger matrix (e.g., sub-graph selection)	SpRef
Assign	Assign to a sub-matrix of a larger matrix (e.g., sub-graph assignment)	SpAsgn
Apply	Apply <i>unary function</i> to each element of matrix (e.g., edge weight modification)	Apply
Reduce	<i>Reduce</i> along columns or rows of matrices (e.g., vertex degree)	Reduce
Transpose	Swaps the rows and columns of a matrix (e.g., reverse directed edges)	Transpose
BuildMatrix	Build a matrix from row, column, value tuples	Sparse
ExtractTuples	Extract the row, column, value tuples from a matrix	Find



Background: Matrix Multiply (MxM)

$$C = A \oplus . \otimes B$$



- The **Semiring** ($\oplus . \otimes$) determines how this computation is carried out.
- Consists of two **Monoids** (**Binary Function** + identity)
 - \otimes , e.g., (multiply, 1)
 - \oplus , e.g., (add, 0)
- These can be user defined, not adhering strictly to Semiring properties

$$c_{i,j} = \sum_{l=1}^k a_{i,l} \times b_{l,j}$$

GraphBLAS Operation: MxM example

$$\mathbf{C} = \mathbf{A} \oplus . \otimes \mathbf{B}$$

- Required:
 - Two input matrices: $A_{M \times K}$ and $B_{K \times N}$
 - One output matrix: $C_{M \times N}$
 - One semiring: $\oplus . \otimes$

GraphBLAS Operation: MxM example

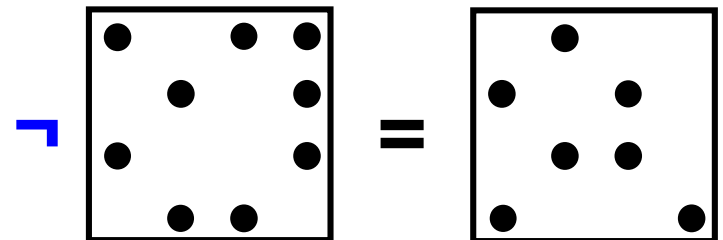
$$\mathbf{C} \oplus = \mathbf{A}^T \oplus \cdot \otimes \mathbf{B}^T$$

- Required:
 - Two input matrices: $A^T_{M \times K}$ and $B^T_{K \times N}$
 - One output matrix: $C_{M \times N}$
 - One semiring: $\oplus \cdot \otimes$
- Optional:
 - Matrix Transpose – only necessary on inputs
 - Accumulation – binary function, can be different from Semiring's \oplus .

GraphBLAS Operation: MxM example

$$\mathbf{C}(\neg\mathbf{M}) \oplus = \mathbf{A}^T \oplus \otimes \mathbf{B}^T$$

- Required:
 - Two input matrices: $\mathbf{A}^T_{M \times K}$ and $\mathbf{B}^T_{K \times N}$
 - One output matrix: $\mathbf{C}_{M \times N}$
 - One semiring: $\oplus \cdot \otimes$
- Optional:
 - Matrix Transpose – only necessary on inputs
 - Accumulation – binary function, can be different from Semiring's \oplus .
 - Output Mask: $\mathbf{M}_{M \times N}$ – specifies which locations in \mathbf{C} can be modified
 - Mask Complement
 - Invert the structure of stored values (sparse)
 - Invert boolean values (dense)



GraphBLAS Operation: MxM example

$$\mathbf{C}(\mathbf{-M}) \oplus = \mathbf{A}^T \oplus \otimes \mathbf{B}^T$$

- Required:
 - Two input matrices: $\mathbf{A}^T_{M \times K}$ and $\mathbf{B}^T_{K \times N}$
 - One output matrix: $\mathbf{C}_{M \times N}$
 - One semiring: $\oplus \cdot \otimes$
- Optional:
 - **Matrix Transpose** – only necessary on inputs
 - **Accumulation** – binary function, can be different from Semiring's \oplus .
 - **Output Mask**: $\mathbf{M}_{M \times N}$ – specifies which locations in C can be modified
 - **Mask Complement**
 - Invert the structure of stored locations (sparse)
 - Invert boolean values (dense)

blue – optional parameters
red – optional modifiers

GBTL Operations

Operation	Math	Out	Inputs
MxM	$\mathbf{C}(\neg\mathbf{M}) \oplus = \mathbf{A}^T \oplus \cdot \otimes \mathbf{B}^T$	C	$\neg, \mathbf{M}, \oplus, \mathbf{A}, \mathbf{T}, \oplus \cdot \otimes, \mathbf{B}, \mathbf{T}$
MxV (VxM)	$\mathbf{c}(\neg\mathbf{m}) \oplus = \mathbf{A}^T \oplus \cdot \otimes \mathbf{b}$	c	$\neg, \mathbf{m}, \oplus, \mathbf{A}, \mathbf{T}, \oplus \cdot \otimes, \mathbf{b}$
EwiseMult	$\mathbf{C}(\neg\mathbf{M}) \oplus = \mathbf{A}^T \otimes \mathbf{B}^T$	C	$\neg, \mathbf{M}, \oplus, \mathbf{A}, \mathbf{T}, \otimes, \mathbf{B}, \mathbf{T}$
EwiseAdd	$\mathbf{C}(\neg\mathbf{M}) \oplus = \mathbf{A}^T \oplus \mathbf{B}^T$	C	$\neg, \mathbf{M}, \oplus, \mathbf{A}, \mathbf{T}, \oplus, \mathbf{B}, \mathbf{T}$
Reduce (row)	$\mathbf{c}(\neg\mathbf{m}) \oplus = \oplus_j \mathbf{A}^T(:,j)$	c	$\neg, \mathbf{m}, \oplus, \mathbf{A}, \mathbf{T}, \oplus$
Apply	$\mathbf{C}(\neg\mathbf{M}) \oplus = f(\mathbf{A}^T)$	C	$\neg, \mathbf{M}, \oplus, \mathbf{A}, \mathbf{T}, f$
Transpose	$\mathbf{C}(\neg\mathbf{M}) \oplus = \mathbf{A}^T$	C	$\neg, \mathbf{M}, \oplus, \mathbf{A} (\mathbf{T})$
Extract	$\mathbf{C}(\neg\mathbf{M}) \oplus = \mathbf{A}^T(i,j)$	C	$\neg, \mathbf{M}, \oplus, \mathbf{A}, \mathbf{T}, i, j$
Assign	$\mathbf{C}(\neg\mathbf{M}) (i,j) \oplus = \mathbf{A}^T$	C	$\neg, \mathbf{M}, \oplus, \mathbf{A}, \mathbf{T}, i, j$
BuildMatrix	$\mathbf{C}(\neg\mathbf{M}) \oplus = \mathbb{S}^{m \times n}(i,j,v,\oplus)$	C	$\neg, \mathbf{M}, \oplus, \oplus, m, n, i, j, \mathbf{v}$
ExtractTuples	$(i,j,v) = \mathbf{A}(\neg\mathbf{M})$	i,j,v	$\neg, \mathbf{M}, \mathbf{A}$

Notation: i,j – index arrays, v – scalar array, m – 1D mask, **other bold-lower** – vector (column), **M** – 2D mask, **other bold-caps** – matrix, **T** – transpose, \neg - structural complement, \oplus monoid/binary function, $\oplus \cdot \otimes$ semiring, **blue** – optional parameters, **red** – optional modifiers



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Objects

- Index and IndexArray
- Matrices and vectors
 - Structure and values
 - Sparse (and dense, but not today)
- Modifiers
 - Structural Complement (and Masks)
 - Transpose
- Mathematical operations
 - Binary functions vs. Monoids
 - Semirings
 - Accumulation (just another binary function)



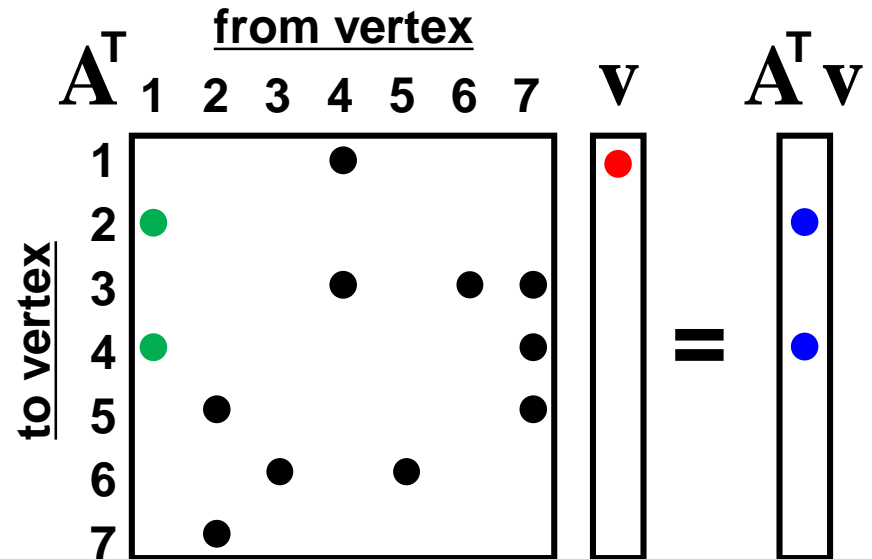
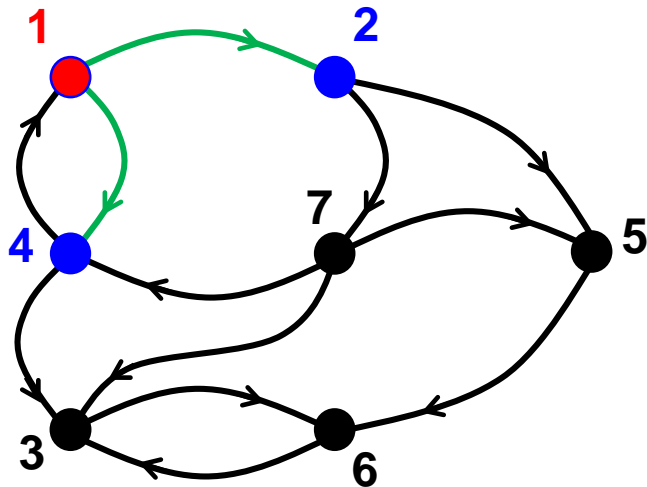
Indices

- **Index** – a value used to locate a position in vectors or matrices (pair of indices).
- **IndexArray** – multiple values used to locate multiple positions

```
// Some typedefs to give GraphBLAS names to some concepts
typedef uint64_t          IndexType;
typedef std::vector<IndexType> IndexArrayType;
```



Sparse Matrices and Vectors



Matrices (e.g. adjacency or incidence)

- Adjacency matrices:
 - “Stored values” – edge
 - “Structural zeros” – no edge/storage
- Two index arrays for positions (structure)
- Scalar array for edge attribute (values)

Vectors (e.g. wavefronts)

- Current: (m x 1) or (1 x n) matrices
- In progress: a different object
 - More efficient storage managing only one index dimension
 - Is an implicit orientation necessary (column in mathematics)?

Achieving Opaque Matrices

- Frontend: Interface here, type/storage details are opaque

```
// Variadic template parameters provide hints for backend matrix type
```

```
template <typename ScalarT, typename... TagST>
```

```
class Matrix
```

```
{
```

```
public:
```

```
    // construct an empty matrix with immutable dimensions
```

```
    Matrix(IndexType num_rows, IndexType num_cols);
```

```
    // Interface, forwards calls to backend
```

```
    IndexType get_nnz() const
```

```
{ return m_matrix.get_nnz(); }
```

```
    IndexType get_nrows() const
```

```
{ return m_matrix.get_nrows(); }
```

```
    IndexType get_ncols() const
```

```
{ return m_matrix.get_ncols(); }
```

```
    //...
```

```
private:
```

```
    detail::matrix_type_generator::result<
```

```
        ScalarT,
```

```
        detail::SparsenessCategoryTag,
```

```
        detail::DirectednessCategoryTag,
```

```
        TagST...>::type
```

```
        m_mat;
```

```
    // template metaprogramming
```

```
    // to select backend type
```

```
    // at compile time.
```

```
    friend void template<...> mxm(...);
```

```
    // all frontend ops are friends
```

```
};
```



Achieving Opaque Matrices (what the user writes)

- Frontend Matrix construction:
 - User provides **hints (tags)** through template parameter packing,
 - backend can make decisions based on hints.

```
Matrix<double, DenseMatrixTag, DirectedMatrixTag>  
    matrix(num_rows, num_columns);
```

- “Sparseness” and “directedness” hints currently implemented.
 - Future support could include layout hints like “fast-column access”
- Backend Matrix classes:
 - Specialized for hardware and implementation
 - Does not have to support or adhere to hints

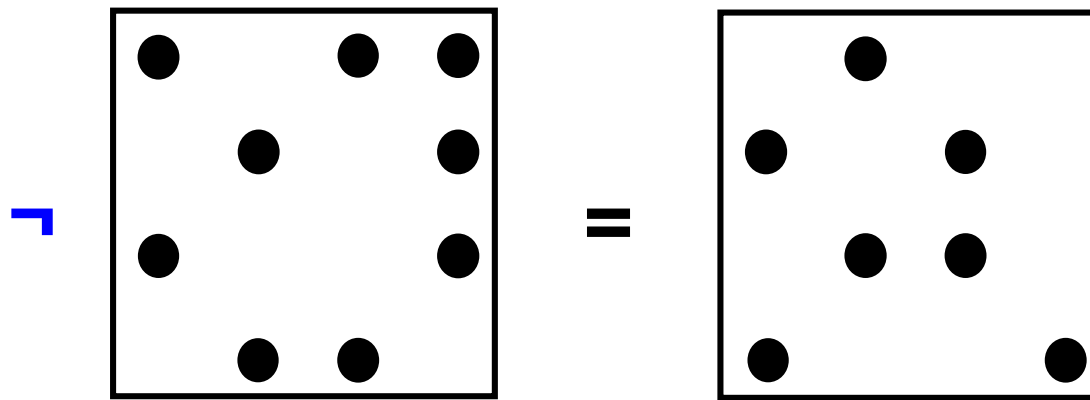


Modifiers

- For two cases
 - Mask complement
 - Matrix transpose
- Purpose:
 - Reduce the number of parameters in operation signatures.
 - Improve readability/usability
- Lightweight wrappers for certain input arguments
- Non-mutating
 - Does not proscribe creation of temporaries



Masks and Structural Complement (WIP)



- Current:
 - Masks also implemented as matrices
 - Values = don't care
 - Complement changes structure (structural zeros become "1")
- Future: a different object
 - Potential: more efficient storage with no scalar values

Complement Modifier (WIP)

- Currently called NegateView (bad name! – will change)
- Templated operations will accept either Matrix or Mask.

```
template <typename MaskT>
class NegateView
{
public:
    NegateView(MaskT const &mask) : m_mask(mask) {}
    IndexType get_nnz() const {
        return m_mask.get_nrows()*m_mask.get_ncols() - m_mask.get_nnz(); }
    //...
private:
    MaskT const &m_mask;
};
```

- Corresponding operation returns a wrapped (*backend*) mask.

```
template<typename MaskT>
NegateView<MaskT> negate(MaskT const &m) {
    return NegateView<MaskT>(backend::negate(m.m_mat));
}
```



Transpose Modifier

- Effectively swaps row and column access.

```
template <typename MatrixT>
class TransposeView
{
public:
    TransposeView(MatrixT const &matrix) : m_matrix(matrix) {}

    IndexType get_nrows() const { return m_matrix.get_ncols(); }
    IndexType get_ncols() const { return m_matrix.get_nrows(); }
    // ...
private:
    MatrixT const &m_matrix;
};
```

- Corresponding operation returns a wrapped (*backend*) matrix:

```
template<typename MatrixT>
TransposeView<MatrixT> transpose(MatrixT const &A) {
    return TransposeView<MatrixT>(backend::transpose(A.mat));
}
```



Mathematical Operations: \oplus , \otimes , $\oplus=$, $\oplus.\otimes$

- Binary functions on multiple domains: $D1 \times D2 \rightarrow D3$

```
template <typename ResultT, typename Arg1T=ResultT, typename Arg2T=ResultT>
struct ArithmeticMultiplyFunc
{
    ResultT operator()(Arg1T const &lhs, Arg2T const &rhs) const {
        return static_cast<ResultT>(lhs) * static_cast<ResultT>(rhs); }
};
```


Mathematical Operations: \oplus , \otimes , $\oplus=$, $\oplus.\otimes$

- Binary functions on multiple domains: $D1 \times D2 \rightarrow D3$
- Monoids (if needed), include the identity for the operation

```
template <typename ResultT, typename Arg1T=ResultT, typename Arg2T=ResultT>
struct ArithmeticAddMonoid
{
    ResultT operator()(Arg1T const &lhs, Arg2T const &rhs) const {
        return static_cast<ResultT>(lhs) + static_cast<ResultT>(rhs);
    }

    ResultT identity() { return static_cast<ResultT>(0) };
};
```



Mathematical Operations: \oplus , \otimes , $\oplus=$, $\oplus.\otimes$

- Binary functions on multiple domains: $D1 \times D2 \rightarrow D3$
- Monoids (if needed), include the identity for the operation
- Semirings for matrix multiply
 - Two binary functions (and identities, if needed)
 - Addition monoid is defined on one domain: $D3 \times D3 \rightarrow D3$

```
template <typename ResultT, typename Arg1T=ResultT, typename Arg2T=ResultT>
struct ArithmeticSemiring
{
    // Additive Monoid
    ResultT zero() const { return static_cast<ResultT>(0); }
    ResultT add(ResultT const &lhs, ResultT const &rhs) const {
        return (lhs + rhs); }

    // Multiplicative Monoid
    ResultT one() const { return static_cast<ResultT>(1); }
    ResultT mult(Arg1T const &lhs, Arg2T const &rhs) const {
        return static_cast<ResultT>(lhs) * static_cast<ResultT>(rhs); }
};
```



Accumulation: \oplus , \otimes , $\oplus=$, $\oplus.\otimes$

- Select one of two binary functions

```
// No accumulation (select rhs), used as the default parameter value
template <typename ResultT>
struct Assign
{
    ResultT operator()(ResultT lhs, ResultT rhs) { return rhs; }
};
```

```
// Accumulation: using arithmetic addition as the default
template <typename ResultT,
         typename BinaryFuncT = ArithmeticAddFunc<ResultT> >
struct Accum
{
    ResultT operator()(ResultT lhs, ResultT rhs) {
        return BinaryFuncT()(lhs, rhs);
    }
};
```



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GBTL Signatures: MxM

$$C(\neg M) \oplus = A^T \oplus \otimes B^T$$

```
template<typename AMatrixT,
        typename BMatrixT,
        typename CMatrixT,
        typename MaskT,
        typename SemiringT,
        typename AccumT    = math::Assign<typename CMatrixT::ScalarType> >
void mxmMasked(AMatrixT const &A,
              BMatrixT const &B,
              CMatrixT      &C,
              MaskT      const &M,
              SemiringT      sr,
              AccumT      accum = AccumT())
{
    same_dimension_check(C, M, std::string("mxmMasked"));
    multiply_dimension_check(A, B, C, std::string("mxmMasked"));
    backend::mxmMasked(A.m_mat, B.m_mat, C.m_mat, M.m_mat, sr, accum);
}
```



GBTL Signatures: MxM

$$C(-M) \oplus = A^T \oplus \otimes B^T$$

```
template<typename AMatrixT,  
        typename BMatrixT,  
        typename CMatrixT,  
        typename MaskT,  
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        typename AccumT = math::Assign<typename CMatrixT::ScalarType> >  
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    backend::mxmMasked(A.m_mat, B.m_mat, C.m_mat, M.m_mat, sr, accum);
}

```



GBTL Signatures: MxM

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GBTL Signatures: MxM

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    backend::mxmMasked(A.m_mat, B.m_mat, C.m_mat, M.m_mat, sr, accum);
}

```



GBTL Signatures: MxM

$$C(\neg M) \oplus = A^T \oplus \cdot \otimes B^T$$

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GBTL Signatures: MxM

$$C(-M) \oplus = A^T \oplus \cdot \otimes B^T$$

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    backend::mxmMasked(A.m_mat, B.m_mat, C.m_mat, M.m_mat, sr, accum);  
}
```



GBTL Signatures: MxM

Modifiers appear at the call site.

$$C(-M) \oplus = A^T \oplus . \otimes B^T$$

```
template<typename AMatrixT,  
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        typename CMatrixT,  
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}
```

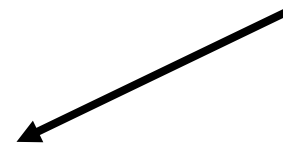


GBTL Signatures: MxM

$$C(-M) \oplus = A^T \oplus \otimes B^T$$

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void mxmMasked(AMatrixT const &A,  
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              CMatrixT      &C,  
              MaskT      const &M,  
              SemiringT      sr,  
              AccumT      accum = AccumT())  
{  
    same_dimension_check(C, M, std::string("mxmMasked"));  
    multiply_dimension_check(A, B, C, std::string("mxmMasked"));  
    backend::mxmMasked(A.m_mat, B.m_mat, C.m_mat, M.m_mat, sr, accum);  
}
```

Simple code like
dimension checks in
the frontend



GBTL Signatures: MxM

$$C(\neg M) \oplus = A^T \oplus \otimes B^T$$

```
template<typename AMatrixT,  
        typename BMatrixT,  
        typename CMatrixT,  
        typename MaskT,  
        typename SemiringT,  
        typename AccumT    = math::Assign<typename CMatrixT::ScalarType> >  
void mxmMasked(AMatrixT const &A,  
              BMatrixT const &B,  
              CMatrixT      &C,  
              MaskT      const &M,  
              SemiringT      sr,  
              AccumT      accum = AccumT())  
{  
    same_dimension_check(C, M, std::string("mxmMasked"));  
    multiply_dimension_check(A, B, C, std::string("mxmMasked"));  
    backend::mxmMasked(A.m_mat, B.m_mat, C.m_mat, M.m_mat, sr, accum);  
}
```

Forward work
to the backend



GBTL Signatures: EwiseMult

$$C(\neg M) \oplus = A^T \otimes B^T$$

```
template<typename AMatrixT,
         typename BMatrixT,
         typename CMatrixT,
         typename MaskT,
         typename BinaryFunctionT,
         typename AccumT = Assign<typename CMatrixT::ScalarType> >
void ewiseaddMasked(AMatrixT const &A,
                   BMatrixT const &B,
                   CMatrixT      &C,
                   MaskT      const &M,
                   BinaryFunctionT func = BinaryFunctionT(),
                   AccumT      accum = AccumT())
{
    same_dimension_check(A, B, std::string("ewiseadd"));
    same_dimension_check(A, C, std::string("ewiseadd"));
    same_dimension_check(C, M, std::string("ewiseadd"));
    backend::ewiseadd(A.m_mat, B.m_mat, C.m_mat, M.m_mat, func, accum);
}
```



GBTL Signatures: Apply

$$C(\neg M) \oplus = f(A^T)$$

```
template<typename AMatrixT,  
        typename CMatrixT,  
        typename MaskT,  
        typename UnaryFunctionT,  
        typename AccumT = Assign<typename CMatrixT::ScalarType> >  
void applyMasked(AMatrixT const &A,  
                CMatrixT      &C,  
                MaskT          &M,  
                UnaryFunctionT func,  
                AccumT         accum = AccumT())  
{  
    same_dimension_check(A, C, std::string("apply"));  
    same_dimension_check(C, M, std::string("apply"));  
    backend::applyMasked(a.m_mat, c.m_mat, M.m_mat, func, accum);  
}
```



GBTL Signatures: Reduce (rows)

$$\mathbf{c}(-\mathbf{m}) \oplus = \bigoplus_j \mathbf{A}^T(:,j)$$

```
template<typename AMatrixT,  
        typename CVectorT,  
        typename MaskT,  
        typename MonoidT = ArithmeticAddMonoid<typename AMatrixT::ScalarType,  
                                                typename BMatrixT::ScalarType,  
                                                typename CMatrixT::ScalarType >,  
        typename AccumT = Assign<typename CMatrixT::ScalarType> >  
void rowReduceMasked(AMatrixT const &A,  
                    CVectorT      &c,  
                    MaskT          &m,  
                    MonoidT        monoid = MonoidT(),  
                    AccumT          accum = AccumT())  
{  
    same_dimension_check(c, m, std::string("rowReduceMasked"));  
    if (A.get_nrows() != c.get_nrows() || c.get_ncols() != 1) {  
        throw graphblas::DimensionException("rowReduceMasked dimension error");  
    }  
    backend::rowReduceMasked(A.m_mat, c.m_mat, m.m_mat, moniod, accum);  
}
```



GBTL Signatures: Extract

$$C(\neg M) \oplus = A^T(i, j)$$

```
template<typename AMatrixT,
        typename CMatrixT,
        typename MaskT,
        typename AccumT = Assign<typename CMatrixT::ScalarType> >
void extractMasked(AMatrixT      const &A,
                  IndexArrayType const &i,
                  IndexArrayType const &j,
                  CMatrixT      &C,
                  MaskT          &M,
                  AccumT         accum = AccumT())
{
    same_dimension_check(C, M, std::string("extract"));
    assign_extract_dimension_check(A, C, i.begin(), j.begin());
    backend::extract(A.m_mat, i, j, C.m_mat, M.m_mat, accum);
}
```



Contents

- Operations Overview
- Object Design
- Operations – Function Signatures
- **Example Algorithm**
- Summary and Future Work



GBTL Algorithm: Multi-front, Level BFS v0

```
template <typename MatrixT>           // MatrixT scalar type: Integer,..
void bfs_level(MatrixT const &graph,  // MxM adjacency matrix {0,1}
               MatrixT      wavefront, // MxR columns init'd with roots {0,1}
               MatrixT      &levels)  // MxR level results for each BFS
{
    using T = typename MatrixT::ScalarType;

    IndexType rows = wavefront.get_nrows();
    IndexType cols = wavefront.get_ncols();
    MatrixT not_visited(rows, cols);
    T current_level = 0;

    while (wavefront.get_nnz() > 0) {
        // Increment and apply current level to all newly visited nodes.
        arithmetic_n<T, Times<T> > apply_level(++current_level);      ++C
        apply(wavefront, levels, apply_level, Accum<T>());             L+= C W

        mxm(wavefront, graph, wavefront,                               W = W |.& A
            IntBooleanSemiring<T>());

        // Remove previously visited vertices from the wavefront       $\bar{V} = (L == 0)$ 
        apply(levels, not_visited, IsZero<T>());                       W =  $\bar{V}$  .& W
        ewisemult(not_visited, wavefront, wavefront, AndFunc<T>());
    }
}
```



GBTL Algorithm: Multi-front, Level BFS v1

```
template <typename MatrixT>           // MatrixT scalar type: Integer,..
void bfs_level(MatrixT const &graph,  // MxM adjacency matrix {0,1}
               MatrixT      wavefront, // MxR columns init'd with roots {0,1}
               MatrixT      &levels)  // MxR level results for each BFS
{
    using T = typename MatrixT::ScalarType;

    IndexType rows = wavefront.get_nrows();
    IndexType cols = wavefront.get_ncols();
    MatrixT not_visited(rows, cols);
    T current_level = 0;

    while (wavefront.get_nnz() > 0) {
        // Increment and apply current level to all newly visited nodes.
        arithmetic_n<T, Times<T> > apply_level(++current_level);      ++C
        apply(wavefront, levels, apply_level, Accum<T>());             L+= C W

        mxm(transpose(graph), wavefront, wavefront,                    W = A^T |.& W
             IntBooleanSemiring<T>());

        // Remove previously visited vertices from the wavefront
        apply(levels, not_visited, IsZero<T>());                       V-bar = (L == 0)
        ewisemult(not_visited, wavefront, wavefront, AndFunc<T>());   W = V-bar .& W
    }
}
```



GBTL Algorithm: Multi-front, Level BFS v2

```
template <typename MatrixT>           // MatrixT scalar type: Integer,..
void bfs_level(MatrixT const &graph,   // MxM adjacency matrix {0,1}
               MatrixT      wavefront, // MxR columns init'd with roots {0,1}
               MatrixT      &levels)   // MxR level results for each BFS
{
    using T = typename MatrixT::ScalarType;

    IndexType rows = wavefront.get_nrows();
    IndexType cols = wavefront.get_ncols();
    MatrixT not_visited(rows, cols);
    T current_level = 0;

    while (wavefront.get_nnz() > 0) {
        // Increment and apply current level to all newly visited nodes.
        arithmetic_n<T, Times<T> > apply_level(++current_level);
        apply(wavefront, levels, apply_level, Accum<T>());

        mxmMasked(transpose(graph), wavefront, wavefront,
                  negate(levels), IntBooleanSemiring<T>());

        // Remove previously visited vertices from the wavefront
        apply(levels, not_visited, IsZero<T>());
        ewisemult(not_visited, wavefront, wavefront, AndFunc<T>());
    }
}
```

$$W(\neg L) = A^T \cdot W$$



GBTL Algorithms: Maximal Independent Set

```
template <typename MatrixT>
void mis(MatrixT const &graph,           // NxN
         MatrixT      &independent_set, // Nx1: !0 indicates node 'in' set.
         double       seed = 0)
{
    graphblas::IndexType rows, cols, r, c;
    graph.get_shape(rows, cols);
    // check dimensions...

    generator.seed(seed); // for SetRandom functor (not shown)

    typedef Matrix<double, /*Tags...*/> RealMatrix;
    using T = typename MatrixT::ScalarType;

    // This will hold the set (non-zero) implies part of the set
    RealMatrix neighbor_max(rows, 1);
    RealMatrix new_members(rows, 1);
    RealMatrix new_neighbors(rows, 1);
    RealMatrix prob(rows, 1);

    RealMatrix candidates(rows, 1); fill(candidates, 1.0);
    RealMatrix degrees(rows, 1);    fill(degrees, 1.0);

    // Compute degree of each node, add 1 to prevent divide by zero
    rowReduce(graph, degrees, ArithmeticAddMonoid<double, T, T>());
}
```



GBTL Algorithms: Maximal Independent Set

```
while (candidates.get_nnz() > 0)
{
    // Assign random values (scaled by degree) to all non-zero candidate elements.
    // Ensures that any ties that occur between neighbors will eventually be
    // broken, and higher degree nodes are more likely selected.
    ewisemult(candidates, degrees, prob, SetRandom());

    // find the neighbor of each source node with the max random number
    mxm(graph, prob, neighbor_max, MaxSelect2ndSemiring<double>());

    // Select source node if its probability is > neighbor_max
    wiseadd(prob, neighbor_max, new_members, GreaterThan<double>());

    // Add new members to independent set.
    wiseadd(independent_set, new_members, independent_set, OrFn<double>());

    // Zero out candidates of new_members selected for independent set
    ewisemult(negate(new_members), candidates, candidates);

    if (candidates.get_nnz() == 0) { break; } // Early exit

    // Neighbors of new members can also be removed
    mxm(graph, new_members, new_neighbors, MaxSelect2ndSemiring<double>());

    // Zero out candidates of new member neighbors
    ewisemult(negate(new_neighbors), candidates, candidates);
}
}
```



Summary and Future Work

- GraphBLAS Template Library
 - Separation of concerns: hardware tuning vs. algorithm design
 - C++ templates
 - Expressive syntax in algorithm development (we could do more).
 - Low overhead (first session)
 - Similar to C API Specification (where possible/reasonable)
- Current and Future Work
 - Tracking C API Specification decisions
 - Multiple domains in monoids and semirings
 - Mask and structural complement
 - Variants of basic operations (reduce to scalar, assign/extract columns)
 - More algorithms
 - GraphBLAS C++ API Specification and Reference Implementation



Questions?



BACKUPS



Background: Semi-Ring, Monoid, Binary Function

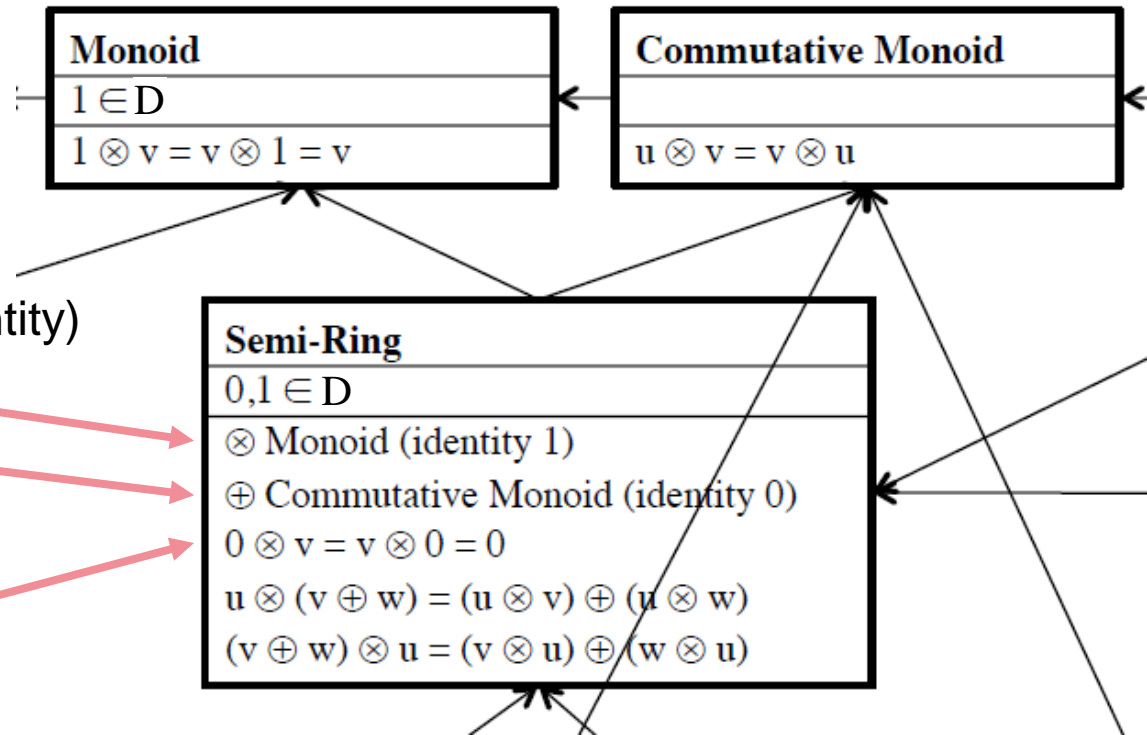
One Domain, D

Two Monoids (binary func, identity)

- \otimes , (“multiply”, “1”)
- \oplus , (“add”, “0”)

Additive identity

= Multiplicative annihilator
= “Structural zero”

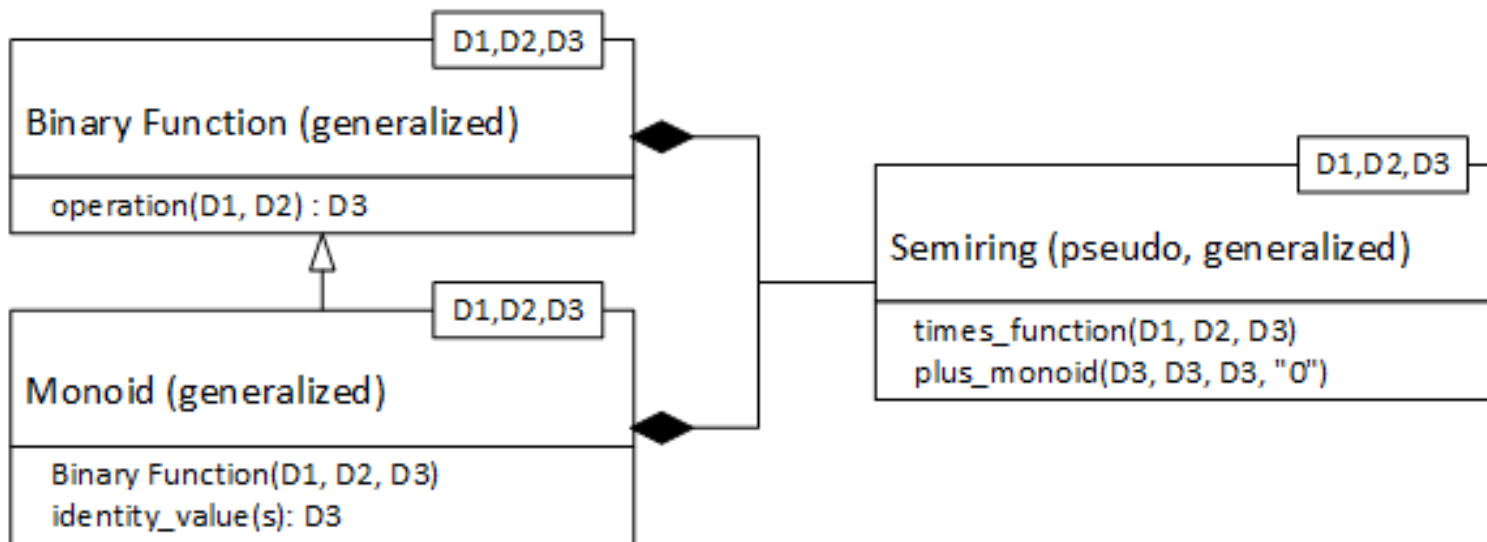


However...

- User can specify arbitrary binary function pairs (still refer to as \otimes and \oplus)
- Operating over multiple domains
- That may not have a multiplicative annihilator

“Generalized” Semiring Design

- Binary multiply function on multiple domains: $D1 \times D2 \rightarrow D3$
- Additive monoid on one domain: $D3 \times D3 \rightarrow D3$
- Additive identity specified on $D3$
- Current discussion:
 - Do we need a “Generalized Monoid” shown here?
 - Do we need to specify the multiplicative identity?
 - Do we require overlapping domains for the additive identity: $0 \in D1 \cap D2 \cap D3$



Transpose Operation

Overloaded for 3 semantics:

1. Return a TransposeView wrapper around a *backend* Matrix.

```
// Modifier
template<typename MatrixT>
TransposeView<MatrixT> transpose(MatrixT const &A) {
    return TransposeView<MatrixT>(backend::transpose(A.mat));
}
```

2. Populate a new matrix with the scalar values transposed.
3. Modify the internal storage of existing matrix

```
// GraphBLAS operation: can check if A and C are the same object,
// and transpose in place if so.
template<typename MatrixT>
void transpose(MatrixT const &A, MatrixT &C) {...}
```

