An Optimal Real-Time Voltage and Frequency Scaling for Uniform Multiprocessors

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Motivation

Chip multiprocessors are the way to deal with increasing computational load in embedded real-time systems

- Power consumption, heat dissipation, and other physical constraints render single processors impractical

Power consumption is a concern in battery-powered real-time systems

- battery life time
- battery weight

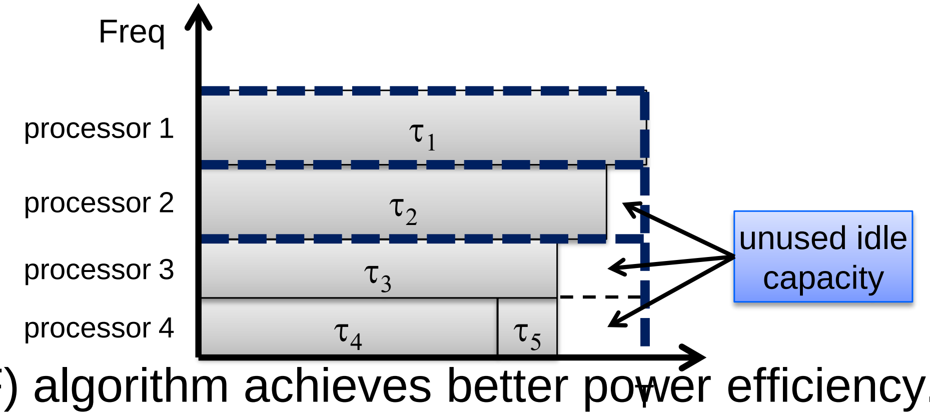
Overview

Voltage and frequency scaling (VFS) allows reducing the power consumption of a processor, and its speed.

VFS in real-time systems must ensure that the system remains schedulable.

Existing VFS algorithms for multiprocessors leave unused idle capacity.

- processor constraints
- algorithm constraints



Growing Minimum Frequency (GMF) algorithm achieves better power efficiency.

- removes algorithm constraint
- reduces impact of processor constraints

Problem Description

Given:

- multiprocessor platform supporting independent VFS, and
- a set of periodic tasks with implicit deadlines

Compute:

- frequency assignment that minimizes power consumption while meeting tasks' deadlines

Processor Power-Frequency Relationship

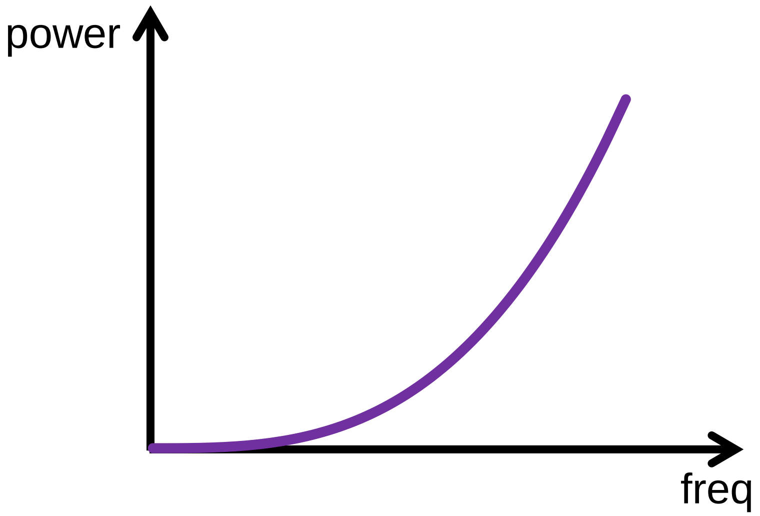
Dynamic power in processors is proportional to the product of the processor frequency and the square of the supply voltage.

$$P \propto V^2 f$$

In VFS, power can be reduced by reducing the frequency, which allows a corresponding reduction in the voltage.

Since voltage is proportional to the frequency we can approximate as

$$P \propto f^3$$



Task and Platform Model

Tasks

n : number of tasks

C_i : execution time of task τ_i, measured at the highest frequency

T_i : period of task τ_i

$D_i = T_i$: implicit deadlines

$u_i = {}^{C_i}/_{T_i}$: utilization of task τ_i

$U = \sum_{i=1}^{n} u_i$: total utilization

Platform:

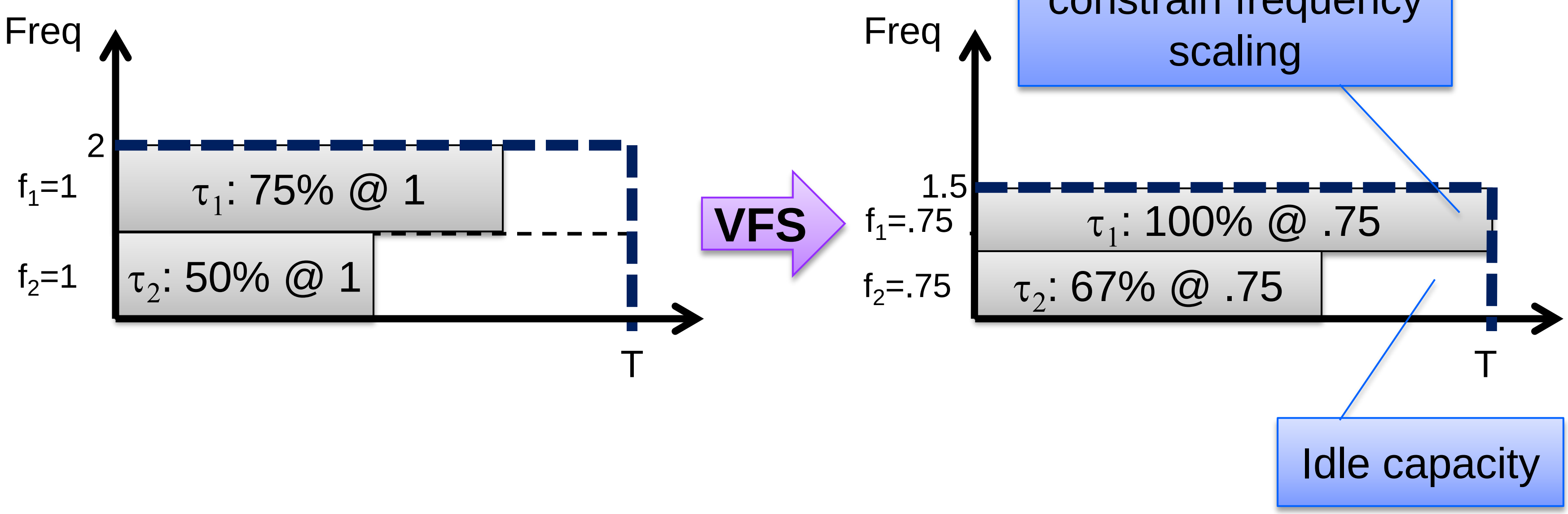
m : number of processors (all identical)

f_i : normalized frequency (1 = highest frequency) for processor *i*

Uniform Frequency Scaling

- All processors assigned the same frequency
- Tasks scheduled with an optimal global scheduler (e.g., LNREF)

$$f = \max\left(U/m, u_1, \ldots, u_n\right)$$



Non-Uniform Frequency Scaling 1

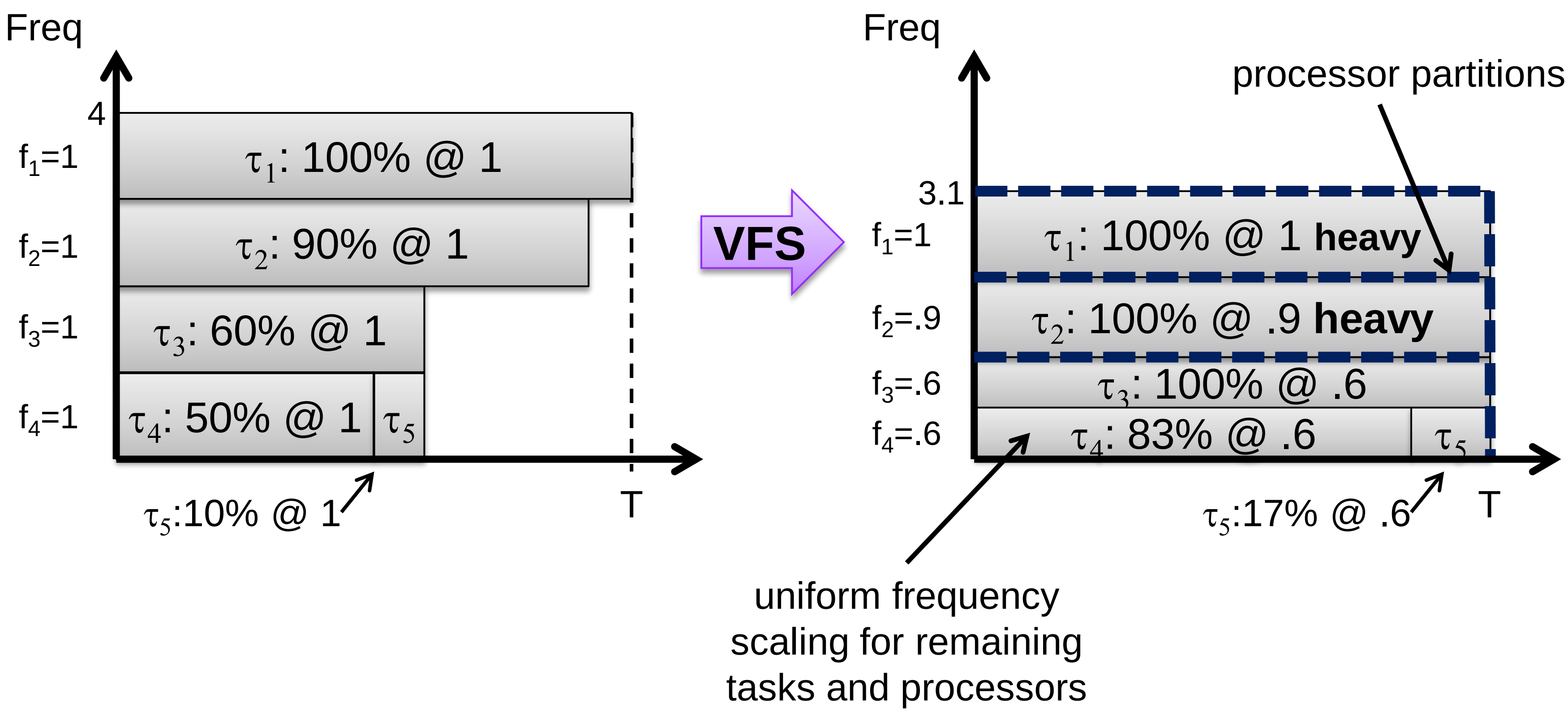
- Processor frequencies are assigned independently

Decide Independent Frequency algorithm avoids heavy task bottleneck [Funaoka 2008]

- Task τ_i is heavy if its utilization would drive up the uniform frequency assignment for the remaining processors, i.e. $u_i > \frac{\sum_{j=i}^{n} u_j}{m-i+1}$
- Each heavy task is assigned its own processor
- Remaining light tasks globally scheduled in remaining processors with uniform frequency assignment

Non-Uniform Frequency Scaling 2

Decide Independent Frequency is optimal if frequency can be scaled continuously (i.e. to any frequency in a range)

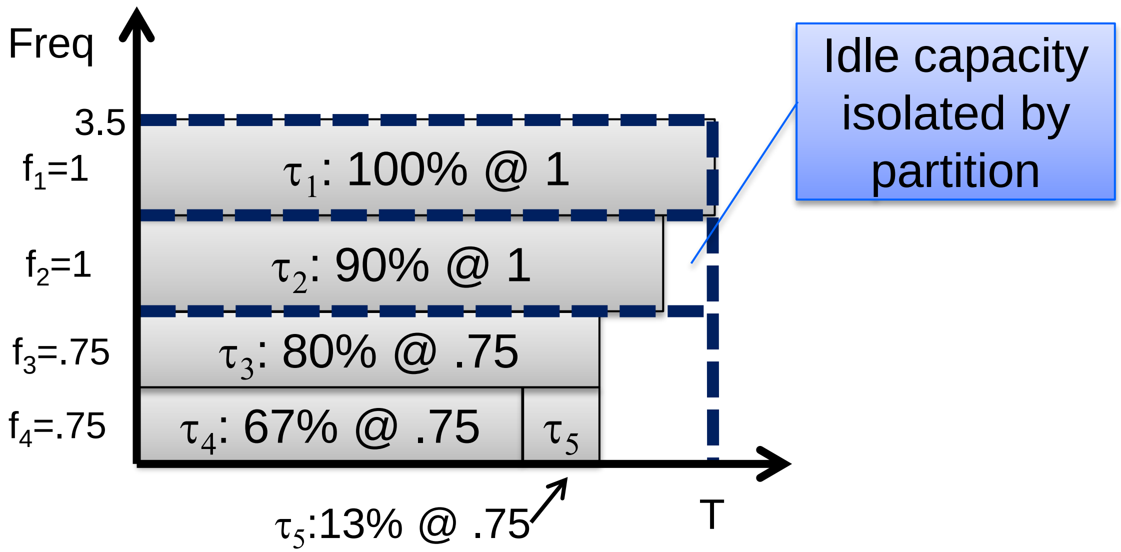


Discrete Frequency Steps

However, processors support a limited number of frequencies.

- DIF is not optimal in that setting
- Computing the optimal partition of processors and the frequency assignment is NP-Hard

Example: supported frequencies: 1, .75, .5



Achieving Better Power Efficiency

Two problems

- Discrete frequency steps force leaving idle capacity in processor partitions
- Unused capacity in a processor partition cannot be used by tasks assigned to other partitions

Observation: if we can optimally schedule tasks allowing them to migrate between processors running at different frequencies we can do better

- avoid the set partition problem (and its computational complexity)
- achieve better power efficiency
  - no fragmentation of platform capacity
  - capacity left by heavy tasks is not wasted

U-LLREF

U-LLREF [Funk 2010] is an optimal global scheduling algorithm for uniform multiprocessors

- an extension of LLREF (a DP-fair algorithm)
- processors can run at different frequencies

A task set is schedulable by U-LLREF on a platform if the following holds

$$\sum_{i=1}^{k} u_i \leq \sum_{i=1}^{k} f_i \qquad \forall k \in \{1, \ldots, m-1\}$$

$$\sum_{i=1}^{n} u_i \leq \sum_{i=1}^{m} f_i$$

where $\boldsymbol{u_1} \geq \cdots \geq \mathbf{u_n}$ and $\boldsymbol{f_1} \geq \cdots \geq \mathbf{f_m}$

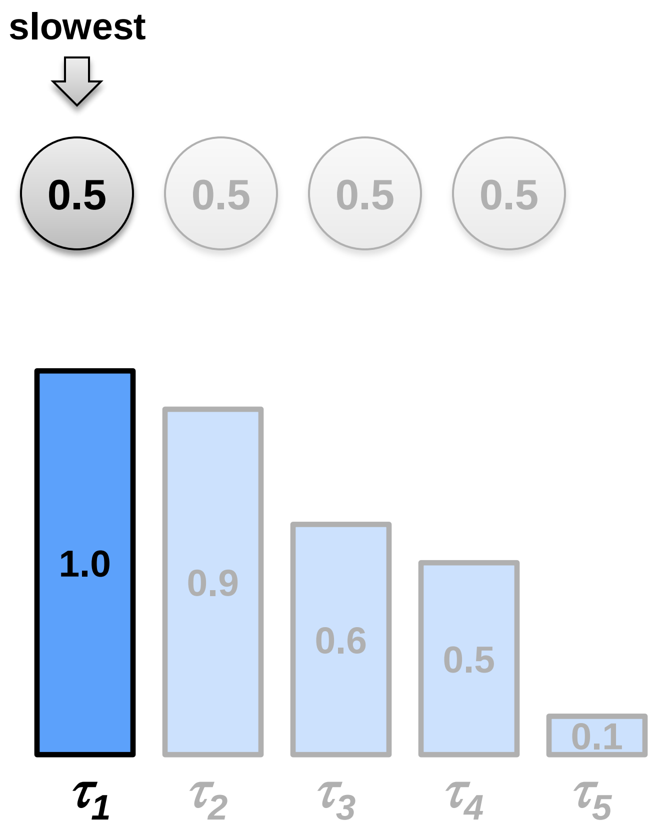
Growing Minimum Frequency Algorithm

Overview: satisfy each condition of the U-LLREF test using the most power efficient assignment of frequencies (lowest possible and distributed as uniformly as possible)

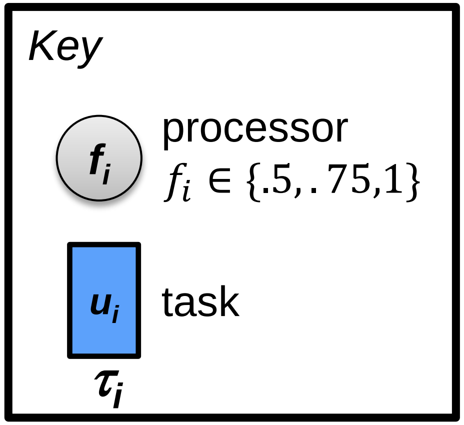
```
assign the lowest frequency to all the processors
for k = 1 to m do
  while kth U-LLREF condition not satisfied do
    increase the frequency of the slowest
       processors in subset 1..k to the next
       frequency step
  end while
end for
```

GMF Example 1

k = 1

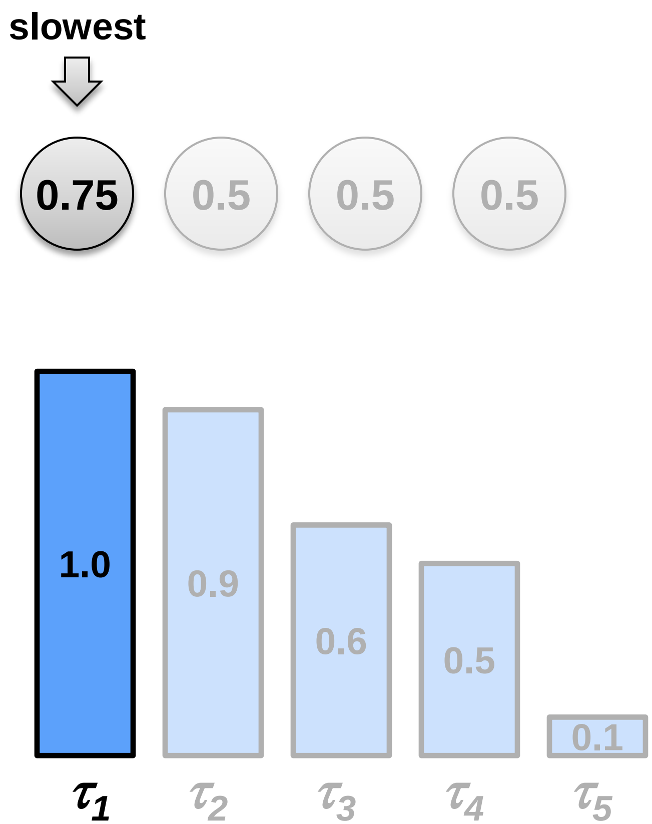


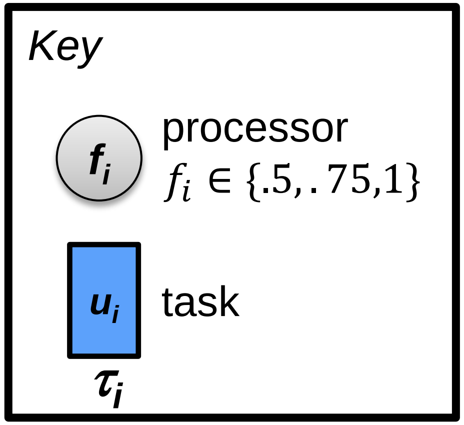
$$\sum_{i=1}^{k} u_i = 1.0 \quad > \quad \sum_{i=1}^{k} f_i = 0.5$$



GMF Example 2

k = 1

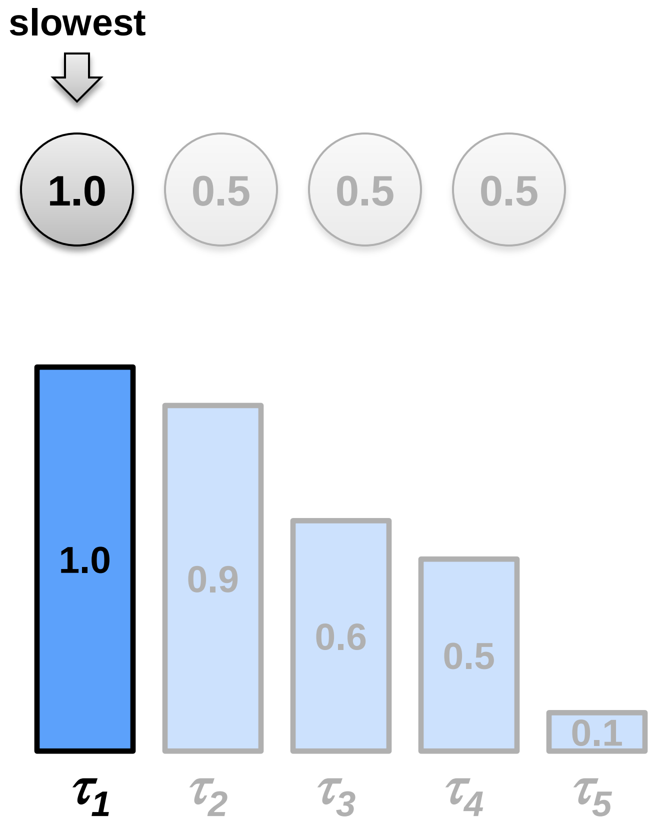




$$\sum_{i=1}^{k} u_i = 1.0 \quad > \quad \sum_{i=1}^{k} f_i = 0.75$$

GMF Example 3

k = 1



Key

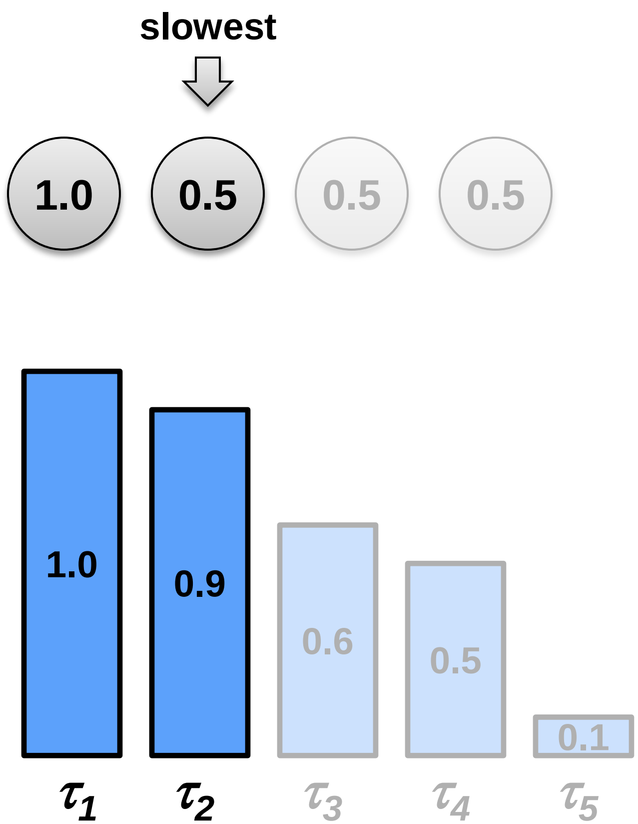
processor $f_i \in \{.5, .75, 1\}$

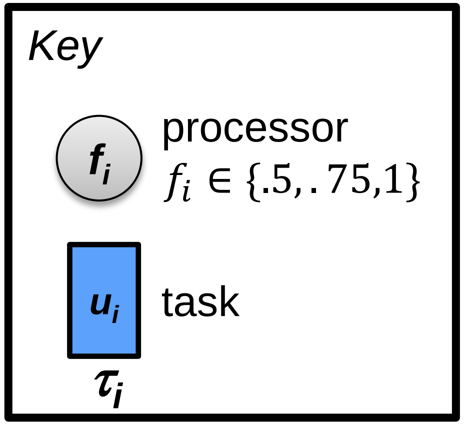
task u_i, τ_i

$$\sum_{i=1}^{k} u_i = 1.0 \quad = \quad \sum_{i=1}^{k} f_i = 1.0$$

GMF Example 4

k = 2

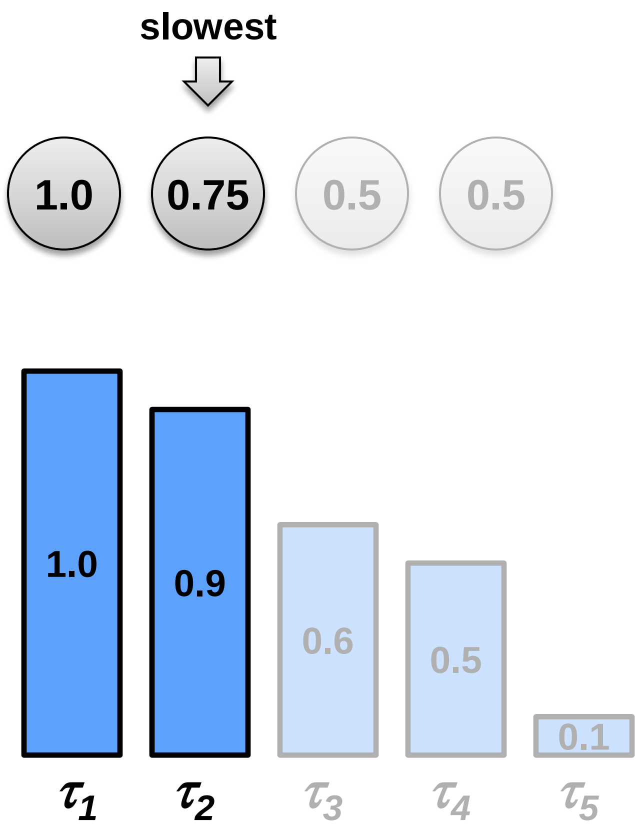


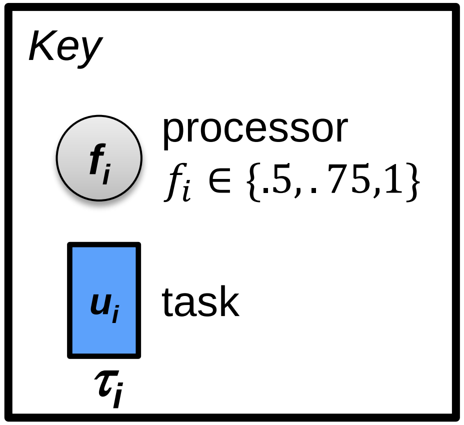


$$\sum_{i=1}^{k} u_i = 1.9 \quad > \quad \sum_{i=1}^{k} f_i = 1.5$$

GMF Example 5

k = 2

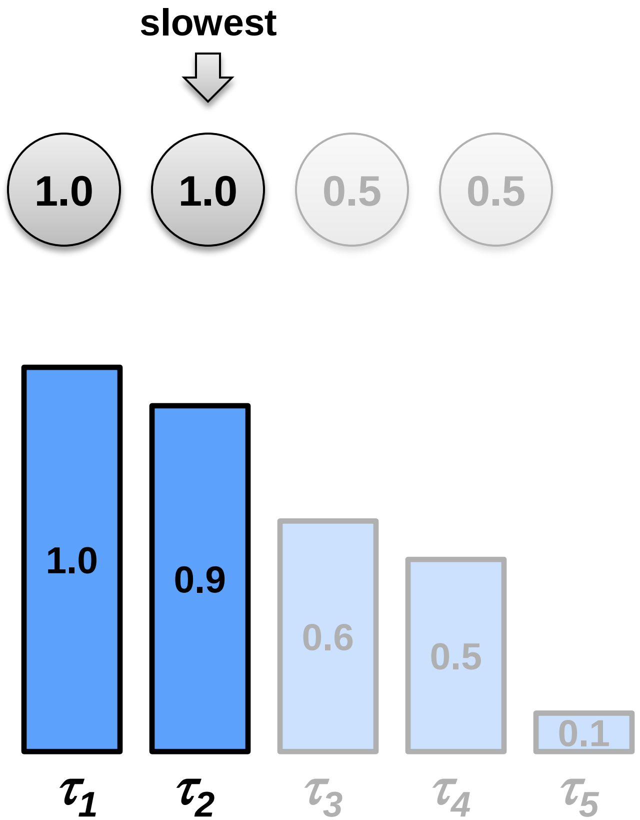




$$\sum_{i=1}^{k} u_i = 1.9 \quad > \quad \sum_{i=1}^{k} f_i = 1.75$$

GMF Example 6

k = 2



Key

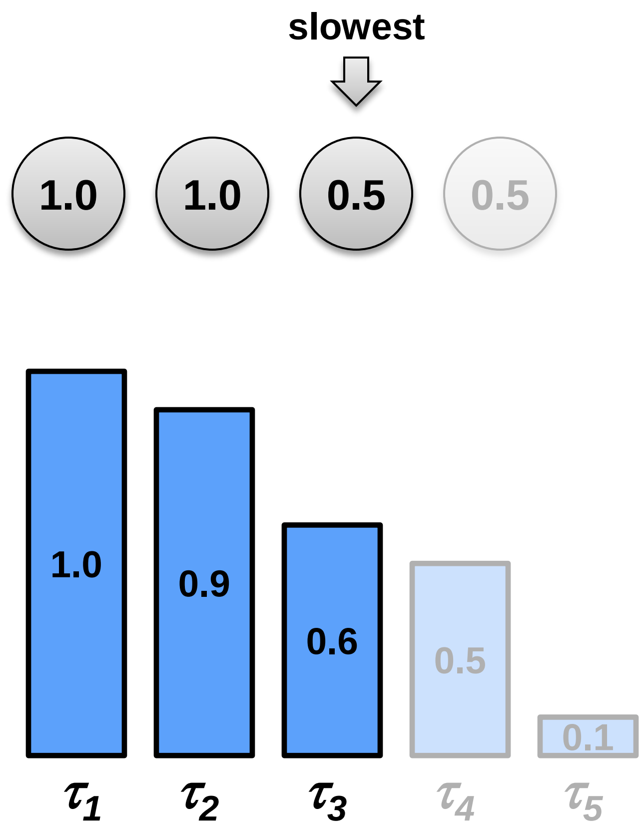
processor $f_i \in \{.5, .75, 1\}$

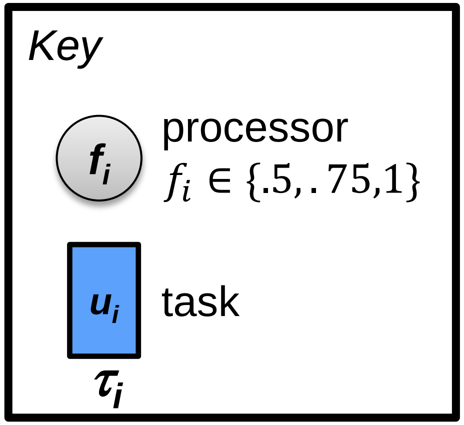
task u_i, τ_i

$$\sum_{i=1}^{k} u_i = 1.9 \quad < \quad \sum_{i=1}^{k} f_i = 2.0$$

GMF Example 6

k = 3

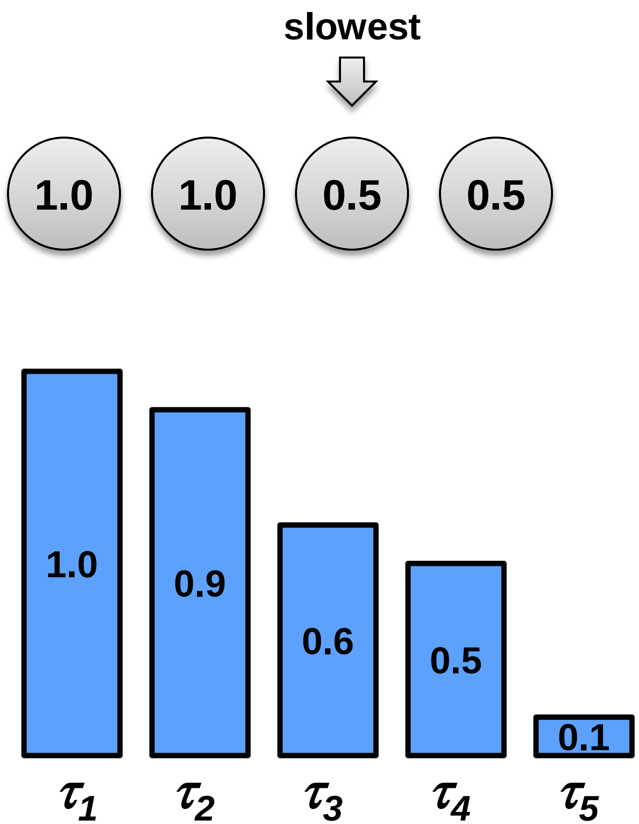


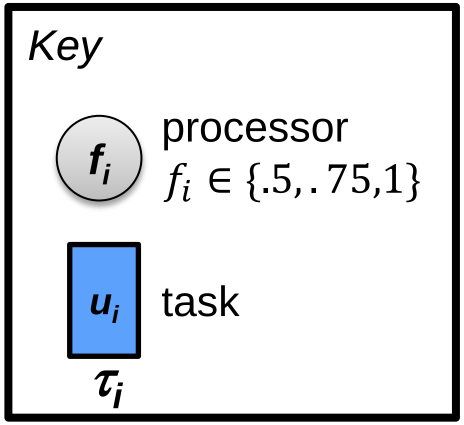


$$\sum_{i=1}^{k} u_i = 2.5 \quad = \quad \sum_{i=1}^{k} f_i = 2.5$$

GMF Example 7

k = 4 = m

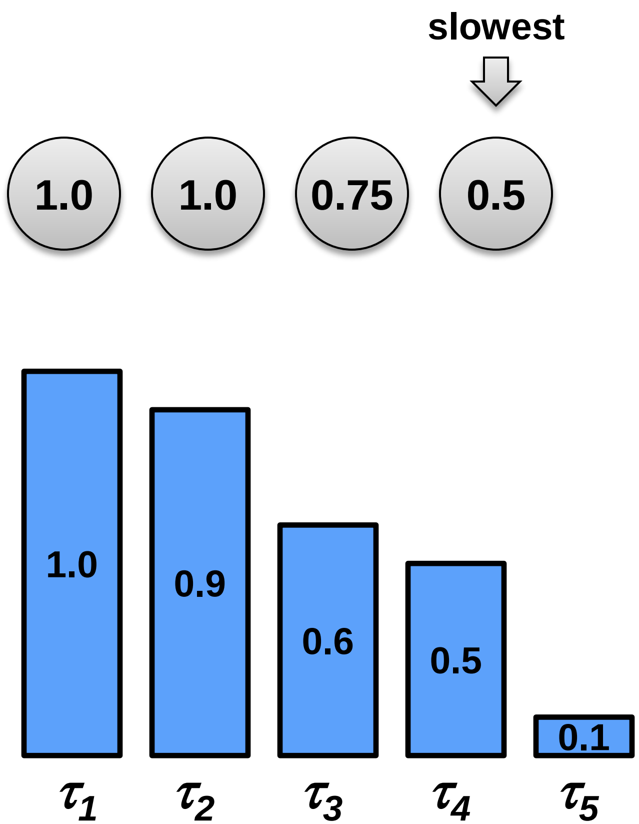


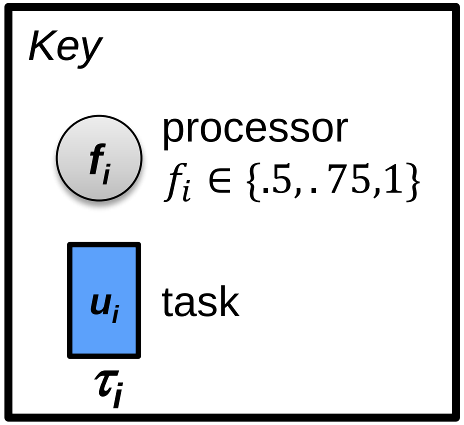


$$\sum_{i=1}^{n} u_i = 3.1 \quad > \quad \sum_{i=1}^{m} f_i = 3.0$$

GMF Example 8

$k = 4 = m$

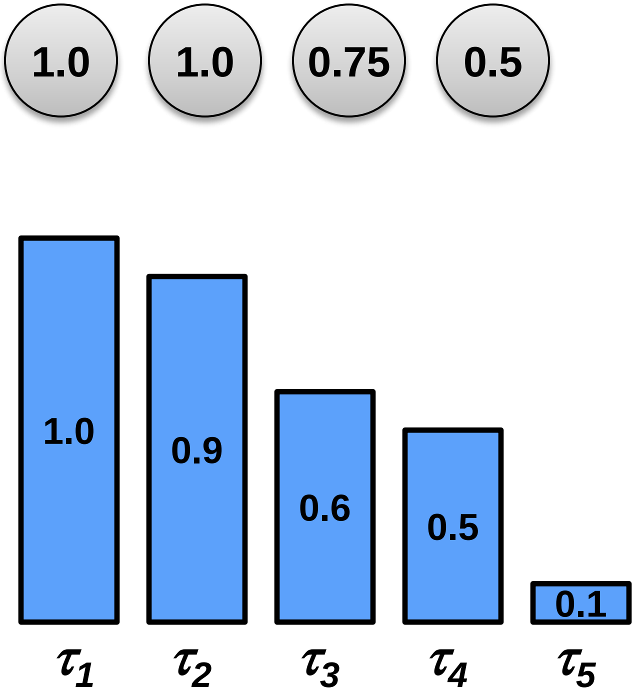




$$\sum_{i=1}^{n} u_i = 3.1 \quad < \quad \sum_{i=1}^{m} f_i = 3.25$$

GMF Example 9

In this case, frequency assignment is the same as in the Exhaustive partition search.



Evaluation

Randomly generated 15,000 tasksets

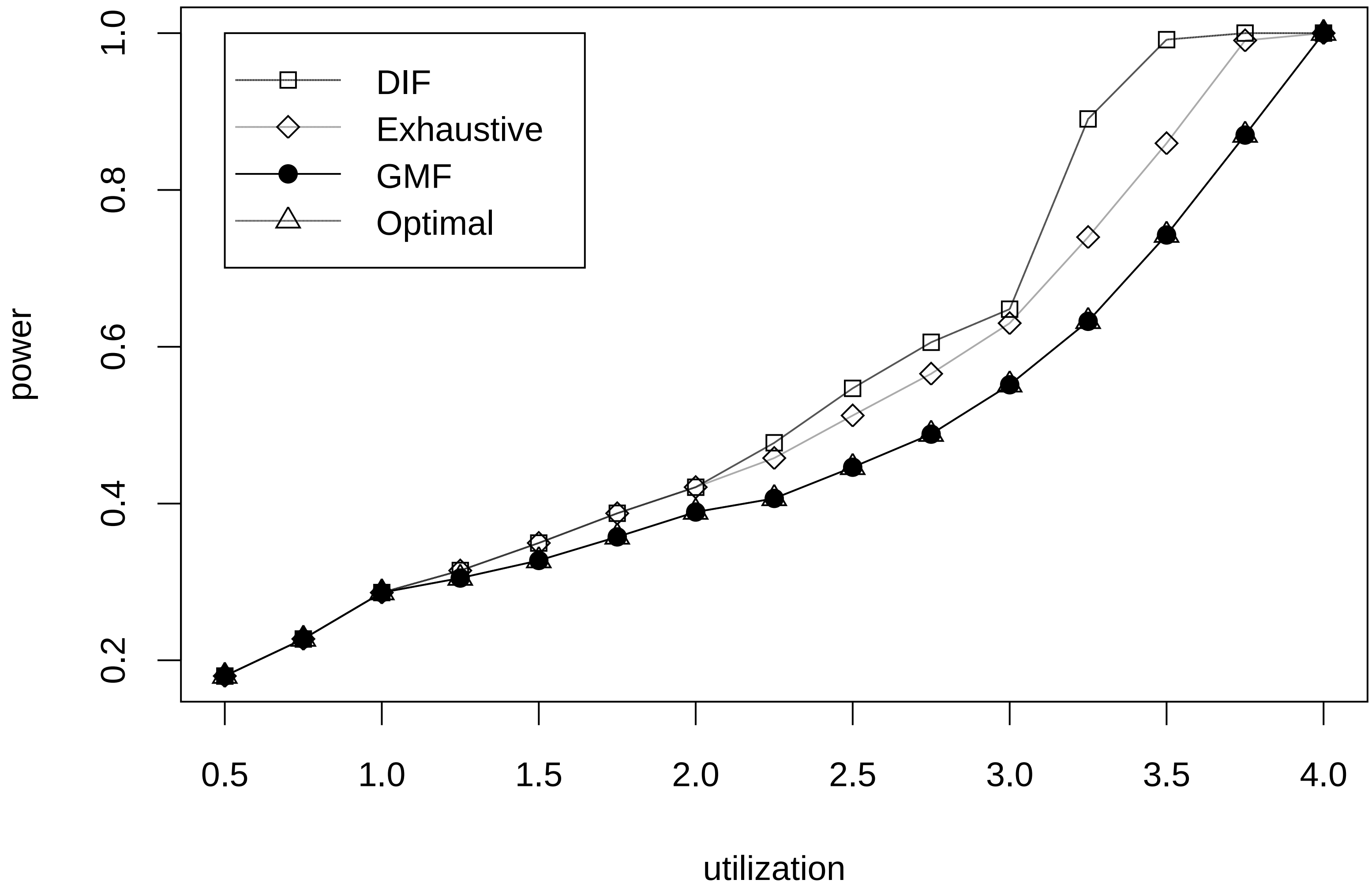
- utilization level ranging from 0.5 to 4 in steps of 0.25
- 1,000 tasksets for each utilization level
- each taskset composed of tasks with random uniform utilization

Used frequencies and voltages of three different quad-core processors

Computed frequency assignment and corresponding power with different multiprocessor VFS algorithms

- Decide Independent Frequency [Funaoka 2008]
- Exhaustive partition/frequency assignment search
- GMF
- Optimal (exhaustive frequency assignment w/o partitions)

Evaluation Results

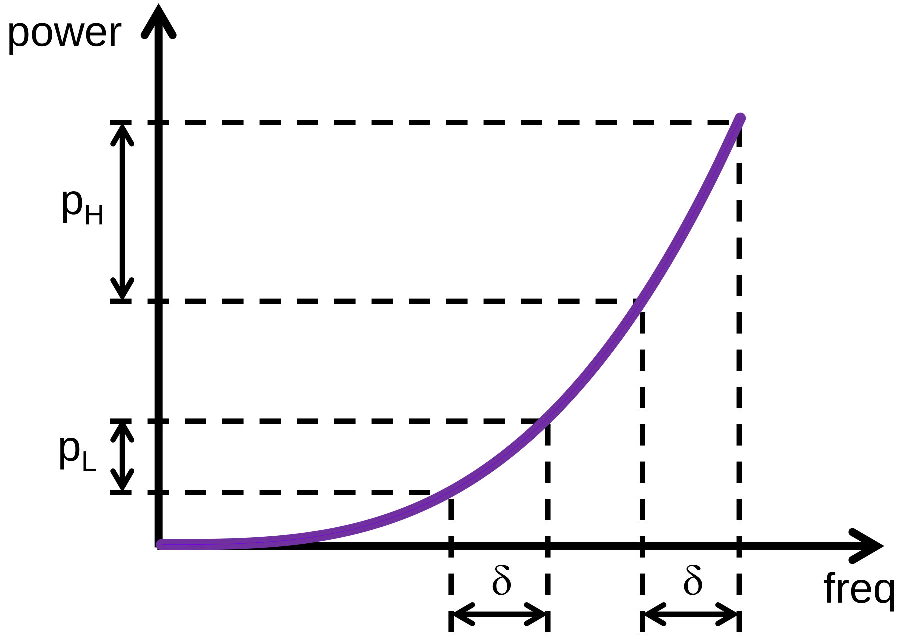


Optimality 1

GMF is optimal when the supported frequency steps are uniform.

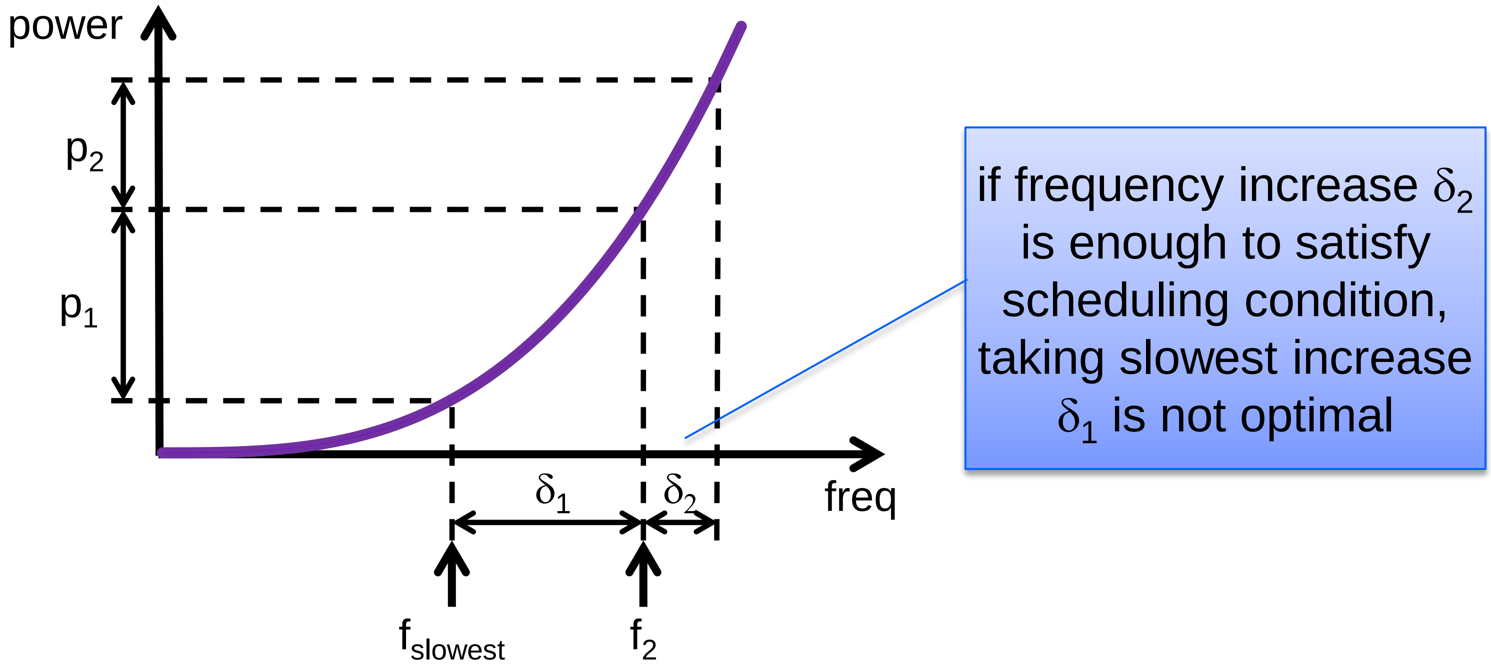
Proof intuition:

- Any frequency step we choose is the same in terms of speed
- Increasing the frequency of the slowest processor requires the smallest power increase
- The optimal frequency assignment for the first *i* conditions bounds from below the optimal assignment for the *i*+1 conditions
- GMF assigns frequencies as even as possible within that bound



Optimality 2

With non-uniform frequency steps, GMF may not optimal



We have observed that for some platforms with non-uniform frequency steps GMF is still optimal

- When the power steps associated frequency steps are non-decreasing

Conclusion

Growing Minimum Frequency (GMF) algorithm computes the optimal frequency assignment to minimize the power consumption of a real-time periodic taskset in a multiprocessor platform.

Evaluation results show up to 30% improvement over previous algorithms.

Avoiding partitioning allows GMF to achieve better power efficiency than optimal partitioned approaches.